$$2\ddot{x} + 3\dot{x} + x = 4 \qquad x(0) = 1 \quad \dot{x}(0) = 1$$

$$2\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + x = 4 \qquad \mathcal{L}[x(t)] = X(s)$$

$$\mathcal{L}\left[2\cdot\frac{d^2x}{dt^2}\right] + \mathcal{L}\left[3\frac{dx}{dt}\right] + \mathcal{L}[x(t)] = 4$$

$$2\cdot\mathcal{L}\left[\frac{d^2x}{dt^2}\right] + 3\mathcal{L}\left[\frac{dx}{dt}\right] + \mathcal{L}[x(t)] = 4$$

$$2\cdot(s^2\cdot X(s) - s - 1) + 3(s\cdot X(s) - 1) + X(s) = 4$$

$$2\cdot s^2\cdot X(s) - 2s - 2 + 3\cdot s\cdot X(s) - 3 + X(s) = 4$$

$$2s^2\cdot X(s) + 3sX(s) + X(s) = 4 + 2s + 2 + 3 =$$

$$X(s)\cdot(2s^2 + 3s + 1) = 9 + 2s$$

$$X(s) = \frac{9 + 2s}{2s^2 + 3s + 1}$$

$$X(s) = \frac{s + 4\frac{1}{2}}{s^2 + 1\frac{1}{2}s + \frac{1}{2}}$$

$$\Delta = b^2 - 4ac$$
$$\Delta = \left(\frac{3}{2}\right)^2 - 4\cdot 1\cdot\frac{1}{2}$$
$$\Delta = \frac{9}{4} - 2$$
$$\Delta = \frac{1}{4}$$

$$X(s) = \frac{s + \frac{9}{2}}{(s+1)(s+\frac{1}{2})}$$

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-\frac{3}{2} - \sqrt{\frac{1}{4}}}{2\cdot 1} = \frac{-2}{2} = -1$$

$$x_2 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-\frac{3}{2} + \sqrt{\frac{1}{4}}}{2\cdot 1} = \frac{-1}{2} = -\frac{1}{2}$$

$$X(s) = \frac{A_1}{(s+1)} + \frac{A_2}{(s+\frac{1}{2})}$$

$$x(t) = A_1\cdot e^{-t} + A_2\cdot e^{-\frac{1}{2}\cdot t}$$

Obliczamy stałe A_1 i A_2 z wzorów Heavisidea

$$A_1 = \frac{s + \frac{9}{2}}{\cancel{(s+1)}(s+\frac{1}{2})}\cdot\cancel{(s+1)}\Bigg|_{s=-1} = \frac{-1 + \frac{9}{2}}{-1 + \frac{1}{2}} = \frac{\frac{7}{2}}{-\frac{1}{2}} = -7$$

$$A_2 = \frac{s + \frac{9}{2}}{(s+1)\cancel{(s+\frac{1}{2})}}\cdot\cancel{(s+\frac{1}{2})}\Bigg|_{s=-\frac{1}{2}} = \frac{-\frac{1}{2} + \frac{9}{2}}{-\frac{1}{2} + 1} = \frac{\frac{8}{2}}{\frac{1}{2}} = 8$$

$$x(t) = -7\cdot e^{-t} + 8\cdot e^{-\frac{1}{2}\cdot t}$$