

$$G(s) = \frac{Y(s)}{U(s)} = \frac{4}{s+1}$$

$$u(t) = 2 \cdot \sin \omega t$$

$$y(t) = ?$$

$$\mathcal{L}[u(t)] = U(s) \quad \mathcal{L}[y(t)] = Y(s)$$

$$U(s) = 2 \cdot \frac{\omega}{s^2 + \omega^2}$$

$$Y(s) = G(s) \cdot U(s)$$

$$Y(s) = \frac{4}{s+1} \cdot \frac{2\omega}{s^2 + \omega^2} = \frac{8\omega}{(s+1)(s+j\omega)(s-j\omega)}$$

$$s^2 + \omega^2 = 0$$

$$Y(s) = \frac{A_1}{(s+1)} + \frac{A_2}{(s+j\omega)} + \frac{A_3}{(s-j\omega)}$$

$$s_1 = j\omega \quad \vee \quad s_2 = -j\omega$$

$$y(t) = A_1 \cdot e^{-t} + A_2 \cdot e^{j\omega t} + A_3 \cdot e^{-j\omega t}$$

$$A_1 = \frac{8\omega}{(s+1)(s+j\omega)(s-j\omega)} \cdot (s+1) \Big|_{s=-1} = \frac{8\omega}{(-1+j\omega)(-1-j\omega)} = \frac{8\omega}{1+j\omega-j\omega+\omega^2} = \frac{8\omega}{1+\omega^2}$$

$$A_2 = \frac{8\omega}{(s+1)(s+j\omega)(s-j\omega)} \cdot (s-j\omega) \Big|_{s=-j\omega} = \frac{8\omega}{(-j\omega+1)(-j\omega-j\omega)} = \frac{8\omega}{(-j\omega+1) \cdot (-j2\omega)} = \frac{8\omega}{-2\omega^2 + j2\omega}$$

$$A_3 = \frac{8\omega}{(s+1)(s+j\omega)(s-j\omega)} \cdot (s-j\omega) \Big|_{s=j\omega} = \frac{8\omega}{(j\omega+1)2j\omega} = \frac{8\omega}{-2\omega^2 + 2j\omega}$$

$$A_1 = \frac{8\omega}{1+\omega^2} \quad A_2 = \frac{4\omega}{-\omega^2 + j\omega} \quad A_3 = \frac{4\omega}{-\omega^2 + j\omega}$$

$$y(t) = \frac{8\omega}{1+\omega^2} \cdot e^{-t} + \frac{4\omega}{(-\omega^2 - j\omega)} \cdot e^{j\omega t} + \frac{4\omega}{(-\omega^2 + j\omega)} \cdot e^{-j\omega t}$$