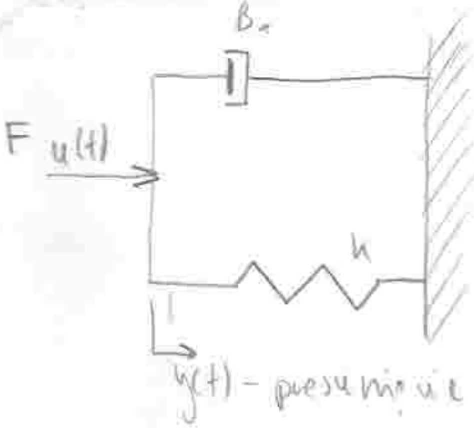


B_1 - stała tłumika wiskotycznego
k - stała sprężyny

$[k] = \frac{N}{m}$

$[B_1] = \frac{N \cdot s}{m}$

Założenia:

$\mathcal{L}[u(t)] = U(s)$

$\mathcal{L}[y(t)] = Y(s)$

$$u(t) = k \cdot y(t) + B_1 \cdot \frac{dy}{dt}$$

transmitancja operatorowa

$$G(s) = \frac{Y(s)}{U(s)}$$

$$U(s) = k \cdot Y(s) + B_1 \cdot s \cdot Y(s)$$

$$U(s) = Y(s)\left[k + B_1 \cdot s\right]$$

$$\frac{B_1}{k} = \tau \quad \left[\frac{N \cdot s}{m} \cdot \frac{m}{N}\right] = [s]$$

$$\frac{Y(s)}{U(s)} = \frac{1}{k + B_1 \cdot s} \quad / \cdot \frac{\frac{1}{k}}{\frac{1}{k}} = \left(\frac{k}{k}\right)^{-1}$$

$$G(s) = \frac{\frac{1}{k}}{(k + B_1 \cdot s) \cdot \frac{1}{k}}$$

$$G(s) = \frac{\frac{1}{k}}{1 + \tau \cdot s}$$

http://www.mbmaster.pl

Transformata Laplace'a

$$F(s) = \int_0^\infty f(t) \cdot e^{-s \cdot t} \cdot dt$$

znaleźć transformatę funkcji $1(t)$

$$1(t) = \begin{cases} 0 \text{ dla } t \leq 0 \\ 1 \text{ dla } t > 0 \end{cases}$$

PODSTAWIENIE

$$F(s) = \int_0^\infty f(t) \cdot e^{-s \cdot t} \cdot dt = \int_0^\infty 1 \cdot e^{-s \cdot t} \cdot dt = \left\{ \begin{array}{c} -s \cdot t = x \\ -s \cdot dt = dx \\ dt = -\frac{dx}{s} \end{array} \right\} =$$

$f(t) = 1$

$$= 1 \cdot \int_0^\infty e^x \cdot \frac{dx}{-s} = -\frac{1}{s} \cdot \int_0^\infty e^x \cdot dx = -\frac{1}{s} \left[e^x \right]_0^\infty =$$

$$= -\frac{1}{s} \cdot \left[e^{-st} \right]_0^\infty = -\frac{1}{s} \cdot [0 - 1] = \frac{1}{s}$$

$$\mathcal{L}[1] = \frac{1}{s}$$

$$a \cdot 1(t) = \frac{d\, a \cdot t(t)}{dt}$$

$$a \cdot t = a \int_0^t (1) t \cdot dt$$

$$\mathcal{L}[a \cdot t] = \mathcal{L} \cdot \left[a \cdot \int_0^t 1(t) dt \right] = a \cdot \mathcal{L}\left[\int_0^t 1(t) \cdot dt \right] =$$

$$= a \cdot \frac{\frac{1}{s}}{s} = \frac{a}{s^2}$$

Transformaty ważniejszych funkcji

Transformata Laplace'a

$$F(s)=\int_0^{+\infty} f(t)\cdot e^{-st}\cdot dt$$

Funkcja	Transformata
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$t^n\cdot e^{\alpha t}$	$\frac{n!}{(s-\alpha)^{n+1}}$
$e^{\alpha t}\cdot\sin\omega t$	$\frac{\omega}{(s-\alpha)^2+\omega^2}$
$e^{\alpha t}\cdot\cos\omega t$	$\frac{s+\alpha}{(s-\alpha)^2+\omega^2}$

Rozwiązać równanie metodą operatorową przy niezerowych warunkach początkowych:

$$2\ddot{x}+3\dot{x}+x=4 \qquad \dot{x}=\frac{dx}{dt}$$

$x(0)=1$

$\dot{x}(0)=1$

$$X(s)=\mathcal{L}[x(t)]$$

$$F(s)=\int_0^{+\infty} f(t)\cdot e^{-st}\cdot dt$$

$$2\frac{d^2x(t)}{dt^2}+3\cdot\frac{dx(t)}{dt}+x(t)=4$$

$$\mathcal{L}\left[2\frac{d^2x}{dt^2}\right]+\mathcal{L}\left[3\frac{dx}{dt}\right]+\mathcal{L}[x(t)]=4$$

$$2\cdot\mathcal{L}\left[\frac{d^2x}{dt^2}\right]+3\cdot\mathcal{L}\left[\frac{dx}{dt}\right]+X(s)=4$$

$$2\cdot\left[s^2\cdot X(s)-s^1\cdot 1-s^0\cdot 1\right]+3\cdot\left[s^1\cdot X(s)-s^0\cdot 1\right]+X(s)=4$$

$$2s^2\cdot X(s)-2s-2+3s\cdot X(s)-3+X(s)=4$$

$$2s^2\cdot X(s)+3s\cdot X(s)+X(s)=4+2s+2+3$$

$$X(s)\cdot\left[2s^2+3s+1\right]=9+2s$$

$$X(s)=\frac{9+2s}{(2s^2+3s+1)}$$

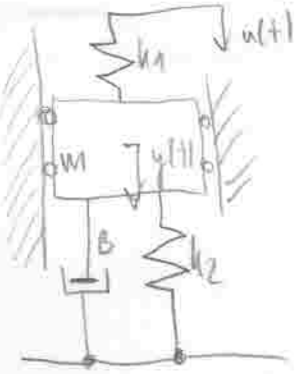
$$X(s)=\frac{2(s+4\tfrac{1}{2})}{2(s^2+\tfrac{3}{2}s+\tfrac{1}{2})}$$

$$X(s)=\frac{s+4\tfrac{1}{2}}{s^2+\tfrac{3}{2}s+\tfrac{1}{2}}$$

$$(s+\quad)(s+\quad)$$

$$F(s)=\frac{1}{(s-2)^2}=\mathcal{L}\left[\tfrac{1}{2}\cdot t\cdot e^{-2\cdot t}\right]$$

$$\mathcal{L}[t^n]=\frac{n!}{s^{n+1}}$$



$\tau = \frac{B}{k_1}\ [s]$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}[u(t)] = U(s)$

$G(s) = \frac{Y(s)}{U(s)}$

WP=0

$$0 = m \cdot \frac{d^2 y(t)}{dt^2} + k_2 \cdot y(t) + B \cdot \frac{dy(t)}{dt} + k_1 \cdot [y(t) - u(t)]$$

$$0 = m \cdot s^2 \cdot Y(s) + k_2 \cdot Y(s) + B \cdot s \cdot Y(s) + k_1 \cdot Y(s) - k_1 \cdot U(s)$$

$$k_1 \cdot U(s) = Y(s) \cdot [m \cdot s^2 + k_2 + B \cdot s + k_1]$$

$k = \left[\frac{kg}{s^2}\right]$

$B = \left[\frac{kg}{s}\right]$

$$\frac{Y(s)}{U(s)} = \frac{k_1}{m \cdot s^2 + k_2 + B \cdot s + k_1}$$

$$G(s) = \frac{k_1}{m \cdot s^2 + k_2 + B \cdot s + k_1} \cdot \frac{\frac{1}{k_1}}{\frac{1}{k_1}}$$

$\frac{B}{k_1} = \tau$

$$G(s) = \frac{1}{\frac{m \cdot s^2}{k_1} + \frac{k_2}{k_1} + \tau \cdot s + 1}$$

$\frac{k_1}{k_1} = 1$

$$G(s) = \frac{1}{\frac{m \cdot s^2}{B} + \frac{k_2 + k_1}{k_1} + \tau \cdot s}$$

$G(s)=\frac{Y(s)}{U(s)}=\frac{4}{s+1}$ $\quad u(t)=2\cdot\sin\omega t$ $\quad y(t)=?$

$\mathcal{L}[u(t)]=U(s) \quad \mathcal{L}[y(t)]=Y(s)$

$U(s)=2\cdot\frac{\omega}{s^2+\omega^2}$

$Y(s)=G(s)\cdot U(s)$

$Y(s)=\frac{4}{s+1}\cdot\frac{2\omega}{s^2+\omega^2}=\frac{8\omega}{(s+1)(s+j\omega)(s-j\omega)}$

$s^2+\omega^2=0$

$Y(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s+j\omega)}+\frac{A_3}{(s-j\omega)}$ $\quad s_1=j\omega \vee s_2=-j\omega$

$y(t)=A_1\cdot e^{-1t}+A_2\cdot e^{-j\omega t}+A_3\cdot e^{j\omega t}$

$A_1=\frac{8\omega}{\cancel{(s+1)}(s+j\omega)(s-j\omega)}\cdot\cancel{(s+1)}\Big|_{s=-1}=\frac{8\omega}{(-1+j\omega)(-1-j\omega)}=\frac{8\omega}{1+j\omega-j\omega+\omega^2}=\frac{8\omega}{1+\omega^2}$

$A_2=\frac{8\omega}{(s+1)\cancel{(s+j\omega)}(s-j\omega)}\cdot\cancel{(s+j\omega)}\Big|_{s=-j\omega}=\frac{8\omega}{(-j\omega+1)(-j\omega-j\omega)}=\frac{8\omega}{(-j\omega+1)\cdot(-j2\omega)}=\frac{8\omega}{-2\omega^2-j2\omega}$

$A_3=\frac{8\omega}{(s+1)(s+j\omega)\cancel{(s-j\omega)}}\cdot\cancel{(s-j\omega)}\Big|_{s=j\omega}=\frac{8\omega}{(j\omega+1)\cdot 2j\omega}=\frac{8\omega}{-2\omega^2+2j\omega}$

$A_1=\frac{8\omega}{1+\omega^2} \quad A_2=\frac{4\omega}{-\omega^2-j\omega} \quad A_3=\frac{4\omega}{-\omega^2+j\omega}$

$y(t)=\frac{8\omega}{1+\omega^2}\cdot e^{-1t}+\frac{4\omega}{(-\omega^2-j\omega)}\cdot e^{-j\omega t}+\frac{4\omega}{(-\omega^2+j\omega)}\cdot e^{j\omega t}$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{4}{s+1} \qquad u(t)=2\cdot\sin\omega t \qquad y(t)=?$$

$$\mathcal{L}[u(t)]=U(s) \qquad \mathcal{L}[y(t)]=Y(s)$$

$$Y(s)=G(s)\cdot U(s)$$

$$U(s)=2\cdot\frac{\omega}{\omega^2+s^2}$$

$$Y(s)=\frac{4}{s+1}\cdot\frac{2\omega}{\omega^2+s^2}=\frac{2\cdot4\omega}{(s+1)(\omega^2+s^2)}=\frac{2\cdot4\omega}{s\omega^2+s^3+\omega^2+s^2}=$$

$$=\frac{8\omega}{s^3+s^2+s\omega^2+\omega^2}=\frac{8\omega}{s(s^2+s+\omega^2)+\omega^2}=\frac{8\omega}{s(s^2+s)+\omega^2(s+1)}$$

$s(s^2+s)-$

$\Delta=1^2-4\cdot a\cdot 0$

$\Delta=1$

$x_1=\frac{-b-\sqrt{\Delta}}{2\cdot a}$

$s_1=-1$

$x_2=\frac{-b+\sqrt{\Delta}}{2\cdot a}$

$\Delta=b^2-4\cdot a\cdot c \qquad \omega^2=\frac{1}{4} \rightarrow \omega=\frac{1}{2}$

$\Delta=0$

$x_1=-\frac{1}{2}$

$x_2=-\frac{1}{2}$

$$Y(s)=\frac{4}{s\cdot(s+\frac{1}{2})(s+\frac{1}{2})+\frac{1}{4}}=\frac{4}{s(s^2+s+\frac{1}{4})+\frac{1}{4}}=$$

$$=\frac{4}{s^3+s^2+\frac{1}{4}\cdot s+\frac{1}{4}}=\frac{4}{s^3+s^2+\frac{1}{4}(s+1)} \qquad \omega^2=\frac{1}{4} \quad \omega=\frac{1}{2}$$

$$Y(s)=\frac{A_1}{s+1}+\frac{A_2\cdot\omega}{\omega^2+s^2}$$

$$A_1=\frac{8\omega}{(s+1)(s^2+\omega^2)}(s+1)\Bigg|_{s=-1}=\frac{8\omega}{1+\omega^2}=\frac{4}{1+\frac{1}{4}}=$$

$$=\frac{4}{\frac{5}{4}}=\frac{4}{1}\cdot\frac{4}{5}=\frac{16}{5}$$

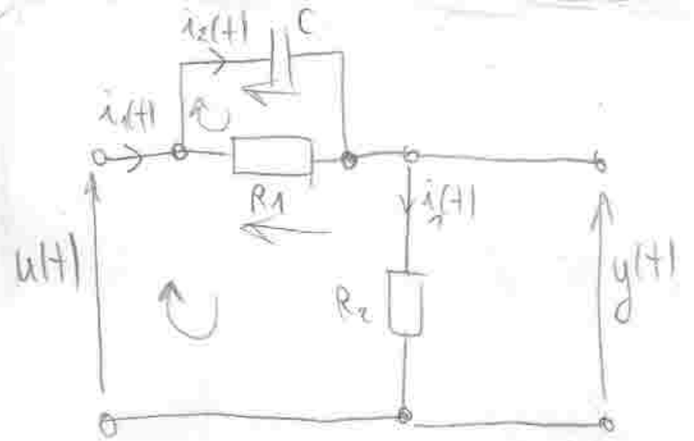
$$A_2=\frac{8\omega\,\cancel{(s^2+\omega^2)}}{(s+1)\cancel{(s^2+\omega^2)}}\Bigg|_{s=i\omega}=\frac{8\cdot\frac{1}{2}}{i\cdot\frac{1}{2}+1}=\frac{4}{i\frac{1}{2}+1}=$$

$$=\frac{4\cdot(1-i\cdot\frac{1}{2})}{(1+i\cdot\frac{1}{2})(1-i\cdot\frac{1}{2})}=\frac{4-2i}{1-\frac{1}{2}i+\frac{1}{2}i+\frac{1}{4}}=$$

$$=(4-2i)\cdot\frac{4}{5}=\frac{16}{5}-\frac{8i}{5}$$

$$y(t)=A_1\cdot e^{-1t}+A_2\cdot\sin\omega t$$

$$y(t)=\frac{16}{5}\cdot e^{-t}+\left(\frac{16}{5}-\frac{8i}{5}\right)\cdot\sin\omega t$$



$\mathcal{L}[u(s)]=U(s)$

$\mathcal{L}[y(s)]=Y(s)$

$\mathcal{L}[i_1(t)]=J_1(s)$

$\mathcal{L}[i_2(t)]=J_2(s)$

$$\begin{cases} u(t)=(i_1(t)-i_2(t)\cdot R_1-i_1(t)\cdot R_2 \\ 0=(i_1(t)-i_2(t))\cdot R_1-\frac{1}{C}\int_0^t i_2(t)\,dt \\ y(t)=i_1(t)\cdot R_2 \end{cases}$$

twierdzenie o całkowaniu

$$\begin{cases} U(s)=J_1(s)\cdot R_1-J_2(s)\cdot R_1-J_1(s)\cdot R_2 & (1) \\ 0=J_1(s)\cdot R_1-J_2(s)\cdot R_2-\frac{1}{C}\cdot\frac{J_2(s)}{s} & (2) \\ Y(s)=J_1(s)\cdot R_2 & (3) \end{cases}$$

(2) $J_1(s)\cdot R_1=J_2(s)\cdot R_2+\frac{1\cdot J_2(s)}{C\cdot s}$ $/:R_1$

$$J_1(s)=J_2(s)\cdot\frac{R_2}{R_1}+\frac{1\cdot J_2(s)}{C\cdot s\cdot R_1}$$

(2) do (3)

$$Y(s)=\left[J_2(s)\cdot\frac{R_2}{R_1}+\frac{J_2(s)}{C\cdot s\cdot R_1}\right]\cdot R_2$$

$$Y(s)=J_2(s)\cdot R_2\cdot\left[\frac{R_2}{R_1}+\frac{1}{C\cdot s\cdot R_1}\right]$$

(2) do (1)

$$U(s)=J_2(s)\cdot\left[\frac{R_2}{R_1}+\frac{1\,J_2(s)}{C\cdot s\cdot R_1}\right]\cdot R_1-J_2(s)\cdot R_1-J_2(s)\cdot\left[\frac{R_2}{R_1}+\frac{1}{C\cdot s\cdot R_1}\right]\cdot R_2$$

$$Y(s) = J_2(s) \cdot \frac{R_2}{R_1} \cdot \left[R_2 + \frac{1}{C \cdot s} \right]$$

$$U(s) = J_2(s) \cdot \left[\left(R_2 + \frac{1}{C \cdot s} \right) - R_1 - \left(\frac{R_2^2}{R_1} + \frac{R_2}{C \cdot s \cdot R_1} \right) \right]$$

$$G(s) = \frac{Y(s)}{U(s)}$$

$G(s)$ - transmitancja operatorowa

$$G(s) = \frac{\cancel{J_2(s)} \cdot \frac{R_2}{R_1} \cdot \left[R_2 + \frac{1}{C s} \right]}{\cancel{J_2(s)} \cdot \left[R_2 + \frac{1}{C \cdot s} - R_1 - \left(\frac{R_2^2}{R_1} + \frac{R_2}{C \cdot s \cdot R_1} \right) \right.}$$

$$G(s) = \frac{\frac{R_2}{R_1} \cdot \left[R_2 + \frac{1}{C \cdot s} \right]}{R_2 + \frac{1}{C \cdot s} - R_1 - \frac{R_2}{R_1} \cdot \left(R_2 + \frac{1}{C \cdot s} \right)}$$

$$G(s) = \frac{\frac{R_2}{R_1} \cdot \left[R_2 + \frac{1}{C \cdot s} \right]}{\frac{R_2 \cdot C \cdot s \cdot R_1}{C \cdot s \cdot R_1} + \frac{1 \cdot R_1}{C \cdot s \cdot R_1} - \frac{R_1 \cdot C \cdot s \cdot R_1}{C \cdot s \cdot R_1} - \frac{R_2 \cdot C \cdot s \cdot R_1}{R_1 \cdot C \cdot s} \left(R_2 + \frac{1}{C \cdot s} \right)}$$

$$G(s) = \frac{\frac{R_2}{R_1} \cdot \left[R_2 + \frac{1}{C \cdot s} \right]}{\frac{R_2 \cdot C \cdot s \cdot R_1}{C \cdot s \cdot R_1} + \frac{R_1}{C \cdot s \cdot R_1} - \frac{R_1 \cdot R_1 \cdot C \cdot s}{C \cdot s \cdot R_1} - \frac{R_2 \cdot R_2 \cdot C \cdot s}{R_1 \cdot C \cdot s} - \frac{R_2 \cdot (C \cdot s)^2}{R_1 \cdot C \cdot s}}$$

Rozwiązać równanie metodą operatorową:

$$2\ddot{x}+3\dot{x}+x=4$$

$x(0)=1$

$\dot{x}(0)=1$

$$\mathcal{L}[x(t)]=X(s)$$

$$\mathcal{L}[1(t)]=\frac{1}{s}$$

$$\mathcal{L}[t^n]=\frac{n!}{s^{n+1}}$$

$$\mathcal{L}[e^{at}]=\frac{1}{s-a}$$

$$\mathcal{L}[t^n\cdot e^{at}]=\frac{n!}{(s-a)^{n+1}}$$

$$2\frac{d^2x(t)}{dt^2}+3\frac{dx(t)}{dt}+x(t)=4$$

$$\mathcal{L}\left[2\frac{d^2x}{dt^2}\right]+\mathcal{L}\left[3\frac{dx}{dt}\right]+\mathcal{L}[x(t)]=4$$

$$2\cdot\mathcal{L}\left[\frac{d^2x}{dt^2}\right]+3\cdot\mathcal{L}\left[\frac{dx}{dt}\right]+\mathcal{L}[x(t)]=4$$

$$2\cdot(s^2\cdot X(s)-s^1\cdot 1-s^0\cdot 1)+3\cdot(s\cdot X(s)-s^0\cdot 1)+X(s)=4$$

$$2\cdot s^2\cdot X(s)-2s-2+3s\cdot X(s)-3+X(s)=4$$

$$2\cdot s^2\cdot X(s)+3s\cdot X(s)+X(s)=4+2+2s+3$$

$$X(s)\cdot(2s^2+3s+1)=9+2s \quad /:(\;)$$

$$X(s)=\frac{9+2s}{2s^2+3s+1}$$

$$X(s)=\frac{s+\frac{9}{2}}{s^2+\frac{3}{2}s+\frac{1}{2}} \rightarrow$$

$$\Delta=b^2-4ac \qquad x_1=\frac{-b-\sqrt{\Delta}}{2a}$$

$$\Delta=\left(\frac{3}{2}\right)^2-4\cdot 1\cdot\frac{1}{2} \qquad x_2=\frac{-b+\sqrt{\Delta}}{2a}$$

$$\Delta=\frac{9}{4}-2$$

$$\Delta=\frac{1}{4}$$

$$s_1=\frac{-\frac{3}{2}-\sqrt{\frac{1}{4}}}{2}=\frac{-\frac{3}{2}-\frac{1}{2}}{2}=-1$$

$$s_2=\frac{-\frac{3}{2}+\sqrt{\frac{1}{4}}}{2}=\frac{-1}{2}=-\frac{1}{2}$$

$$X(s)=\frac{s+\frac{9}{2}}{(s+1)(s+\frac{1}{2})}$$

Aby odnaleźć oryginał trzeba to rozbić na ułamki proste

$$X(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s+\frac{1}{2})}=\frac{A_1\cdot(s+\frac{1}{2})}{(s+1)(s+\frac{1}{2})}+\frac{A_2\cdot(s+1)}{(s+1)(s+\frac{1}{2})}= \rightarrow$$

$$X(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s+\frac{1}{2})}=\frac{A_1(s+\frac{1}{2})}{(s+1)(s+\frac{1}{2})}+\frac{A_2(s+1)}{(s+1)(s+\frac{1}{2})}=$$

$$=\frac{A_1\cdot(s+\frac{1}{2})+A_2(s+1)}{(s+1)(s+\frac{1}{2})}=\frac{A_1\cdot s+A_1\cdot\frac{1}{2}+A_2\cdot s+A_2}{(s+1)(s+\frac{1}{2})}$$

$$X(s)=\frac{s+\frac{9}{2}}{(s+1)(s+\frac{1}{2})}$$

teraz porównujemy liczniki aby znaleźć stałe A_1 i A_2

$$\begin{cases} A_1+A_2=1 \to A_2=1-A_1 \\ \frac{1}{2}A_1+A_2=\frac{9}{2} \end{cases}$$

$$\frac{1}{2}A_1+(1-A_1)=\frac{9}{2}$$

$$\frac{1}{2}A_1+1-A_1=\frac{9}{2}$$

$$\frac{1}{2}A_1-A_1=\frac{9}{2}-\frac{2}{2}$$

$$\frac{1}{2}A_1-A_1=\frac{7}{2}$$

$$-\frac{1}{2}A_1=\frac{7}{2}\quad /\cdot(-2)$$

$$\underline{A_1=-7}$$

$$A_2=1-A_1$$

$$A_2=1-(-7)$$

$$A_2=8$$

$$X(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s+\frac{1}{2})}$$

ORYGINAŁ MA POSTAĆ

$$x(t)=A_1\cdot e^{-t}+A_2\cdot e^{-\frac{1}{2}t}$$

$$\underline{x(t)=-7\cdot e^{-t}+8\cdot e^{-\frac{1}{2}\cdot t}}$$

Wzory Heavisidea

$$A_i=\left.\frac{L(s)\cdot(s-s_i)}{N(s)}\right|_{s=s_i}$$

$$A_1=\left.\frac{(s+\frac{9}{2})\cdot\cancel{(s+1)}}{\cancel{(s+1)}(s+\frac{1}{2})}\right|_{s=-1}=\frac{-1+\frac{9}{2}}{-1+\frac{1}{2}}=\frac{\frac{7}{2}}{-\frac{1}{2}}=\frac{7}{2}\cdot(-2)=-7$$

$$A_2=\left.\frac{(s+\frac{9}{2})\cancel{(s+\frac{1}{2})}}{(s+1)\cancel{(s+\frac{1}{2})}}\right|_{s=-\frac{1}{2}}=\frac{-\frac{1}{2}+\frac{9}{2}}{-\frac{1}{2}+1}=\frac{\frac{8}{2}}{\frac{1}{2}}=\frac{8}{2}\cdot\frac{2}{1}=8$$

$$G(s)=\frac{5s+1}{s^2\cdot(s-3)}$$

$$G(s)=\frac{Y(s)}{U(s)} \rightarrow Y(s)=G(s)\cdot U(s)$$

$$U(s)\rightarrow\boxed{G(s)}\xrightarrow{Y(s)} \quad y(t)=?$$

$$u(t)=5\cdot 1(t)$$

$$\mathcal{L}[u(t)]=\mathcal{L}[5\cdot 1(t)]=5\cdot\mathcal{L}[1(t)]=\frac{5}{s} \rightarrow U(s)=\frac{5}{s}$$

$$\left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$$

$$Y(s)=G(s)\cdot U(s)=\frac{5s+1}{s^2\cdot(s-3)}\cdot\frac{5}{s}=\frac{25s+5}{s^3\cdot(s-3)}$$

$$Y(s)=\frac{A}{(s-3)}+\frac{B_1}{s}+\frac{B_2}{s^2}+\frac{B_3}{s^3}$$

$$y(t)=A\cdot e^{3\cdot t}+B_1\cdot 1(t)+B_2\cdot t+\frac{1}{2}\cdot B_3\cdot t^2$$

$$A=\left.\frac{(25s+5)\cdot\cancel{(s-3)}}{s^3\cdot\cancel{(s-3)}}\right|_{s=3}=\frac{25\cdot 3+5}{3^3}=\frac{80}{27}$$

$$B_3=\left.\frac{(25s+5)\cdot\cancel{s^3}}{\cancel{s^3}\cdot(s-3)}\right|_{s=0}=\frac{25\cdot 0+5}{0-3}=-\frac{5}{3}$$

$$B_2=\frac{1}{1!}\cdot\frac{d}{ds}\left(\frac{25s+5}{(s-3)}\right)\Bigg|_{s=0}=\frac{1}{1!}\cdot\left.\frac{25\cdot(s-3)-(25s+5)\cdot 1}{(s-3)^2}\right|_{s=0}=\frac{-75-5}{9}=-\frac{80}{9}$$

$$B_1=\frac{1}{2!}\cdot\frac{d}{ds}\left(\frac{\cancel{25s}-75-\cancel{25s}-5}{(s-3)^2}\right)\Bigg|_{s=0}=\frac{1}{2}\cdot\left.\frac{0-(-80)\cdot 2\cdot 1}{(s-3)^4}\right|_{s=0}=\frac{1}{2}\cdot\frac{160}{81}=\frac{160}{162}$$

$$y(t)=\frac{80}{27}\cdot e^{3\cdot t}+\frac{160}{162}\cdot 1(t)-\frac{80}{9}\cdot t-\frac{1}{2}\cdot\frac{5}{3}\cdot t^2$$

wyznaczyć oryginał funkcji

$$F(s)=\frac{s^2+2s+1}{(s+3)^3\cdot s^2}$$

$$F(s)=\frac{A_1}{(s+3)}+\frac{A_2}{(s+3)^2}+\frac{A_3}{(s+3)^3}+\frac{B_1}{s}+\frac{B_2}{s^2}$$

$$x(t)=A_1\cdot e^{-3t}+A_2\cdot t\cdot e^{-3\cdot t}+A_3\cdot\frac{1}{2}\cdot t^2\cdot e^{-3t}+B_1\cdot 1(t)+\ t$$

wyznaczamy stałych A_1, A_2, A_3, B_1, B_2

$$\left(g[f(x)]\right)'=g'[f(x)]\cdot f'(x)$$

$$(f(x)\cdot g(x))'=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{(g(x))^2}$$

$$B_p=\frac{L(s)\cdot(s-s_m)^p}{N(s)}\Big|_{s=s_m}$$

$A=$

$$B_{p-j}=\frac{1}{j!}\cdot\frac{d^j}{ds^j}\left(\frac{L(s)\cdot(s-s_m)}{N(s)}\right)\Big|_{s=s_m}$$

$$A_3=\frac{(s^2+2s+1)\cdot\cancel{(s+3)^3}}{\cancel{(s+3)^3}\cdot s^2}\Big|_{s=-3}=\frac{(-3)^2+2\cdot(-3)+1}{(-3)^2}=\frac{9-6+1}{9}=\frac{4}{9}$$

$$A_2=\frac{1}{1!}\cdot\frac{d}{ds}\left(\frac{(s^2+2s+1)\cdot\cancel{(s+3)^3}}{\cancel{(s+3)^3}\cdot s^2}\right)\Big|_{s=-3}=\frac{d}{ds}\left(\frac{s^2+2s+1}{s^2}\right)\Big|_{s=-3}=\frac{d}{ds}\left(1+2\cdot s^{-1}+s^{-2}\right)\Big|_{s=-3}=$$

$$=\left(0+(-2)s^{-2}-2s^{-3}\right)\Big|_{s=-3}=-\frac{2}{(-3)^2}-\frac{2}{(-3)^3}=-\frac{2}{9}-\frac{2}{-27}=$$

$$=-\frac{2}{9}-\left(-\frac{2}{27}\right)=-\frac{2}{9}+\frac{2}{27}=-\frac{6}{27}+\frac{2}{27}=-\frac{4}{27}$$

$$A_1=\frac{1}{2!}\cdot\frac{d}{ds}\left(-2\cdot s^{-2}-2\cdot s^{-3}\right)\Big|_{s=-3}=\frac{1}{2}\cdot\left(4\cdot s^{-3}+6\cdot s^{-4}\right)\Big|_{s=-3}=$$

$$=\frac{1}{2}\cdot\left(\frac{4}{(-3)^3}+\frac{6}{(-3)^4}\right)=\frac{1}{2}\cdot\left(-\frac{4}{27}+\frac{6}{81}\right)=\frac{1}{2}\cdot\left(-\frac{4}{27}+\frac{2}{27}\right)=\frac{1}{2}\cdot\left(-\frac{2}{27}\right)=$$

$$=-\frac{1}{27}$$

$$x(t)=-\frac{1}{27}\cdot e^{-3t}-\frac{4}{27}\cdot t\cdot e^{-3t}+\frac{4}{9}\cdot\frac{1}{2}\cdot t\cdot e^{-3t}$$

$$G(s)=\frac{5s+1}{s^2\cdot(s-3)} \qquad G(s)=\frac{Y(s)}{U(s)}$$

$$u(t)=5\cdot 1(t)$$

$$\mathcal{L}[u(t)]=U(s)=\frac{5}{s}$$

$$Y(s)=G(s)\cdot U(s)$$

$$Y(s)=\frac{5s+1}{s^2\cdot(s-3)}\cdot\frac{5}{s}$$

$$Y(s)=\frac{25s+5}{s^3\cdot(s-3)}$$

$$Y(s)=\frac{A}{(s-3)}+\frac{B_1}{s}+\frac{B_2}{s^2}+\frac{B_3}{s^3}$$

$$y(t)=A\cdot e^{3\cdot t}+B_1\cdot 1(t)+B_2\cdot t+B_3\cdot\frac{1}{2}\cdot t^2$$

$$A=\frac{(25s+5)\cdot\cancel{(s-3)}}{s^3\cdot\cancel{(s-3)}}\Bigg|_{s=3}=\frac{25\cdot 3+5}{3^3}=\frac{80}{27}$$

$$B_3=\frac{(25s+5)\cdot\cancel{s^3}}{\cancel{s^3}\cdot(s-3)}\Bigg|_{s=0}=\frac{25\cdot 0+5}{(0-3)}=-\frac{5}{3}$$

$$B_2=\frac{1}{1!}\cdot\frac{d}{ds}\left(\frac{25s+5}{(s-3)}\right)\Bigg|_{s=0}=25\cdot(s-3)^{-1}-1\cdot(s-3)^{-2}\cdot(25s+5)\Bigg|_{s=0}=$$

$$=\frac{25}{(0-3)}-\frac{(25\cdot 0+5)}{(0-3)^2}=-\frac{25}{3}-\frac{5}{9}=-\frac{75}{9}-\frac{5}{9}=-\frac{80}{9}$$

$$B_1=\frac{1}{2!}\cdot\frac{d}{ds}\left(\frac{25}{(s-3)}-\frac{(25s+5)}{(s-3)^2}\right)\Bigg|_{s=0}=\frac{1}{2}\cdot\left[-1\cdot(s-3)^{-2}\cdot 25\right)-\left(25\cdot(s-3)^{-2}-2\cdot(s-3)^{-3}\cdot(25s+5)\right)\Bigg]\Bigg|_{s=0}=$$

$$=\frac{1}{2}\cdot\left[-\frac{25}{(0-3)^2}-\left(\frac{25}{(0-3)^2}-2\,\frac{25\cdot 0+5}{(0-3)^3}\right)\right]=\frac{1}{2}\cdot\left[-\frac{25}{9}-\frac{25}{9}+\frac{10}{-27}\right]=$$

$$=\frac{1}{2}\left[-\frac{25}{9}-\frac{25}{9}-\frac{10}{27}\right]=\frac{1}{2}\cdot\left[-\frac{75}{27}-\frac{75}{27}-\frac{10}{27}\right]=\frac{1}{2}\cdot\left(-\frac{160}{27}\right)=-\frac{80}{27}$$

$$y(t)=\frac{80}{27}\cdot e^{3t}-\frac{80}{27}\cdot 1(t)-\frac{80}{9}\cdot t-\frac{5}{6}\cdot t^2$$

$$F(s)=\frac{s^2+1}{s\cdot(s+1)(s-2)}$$ $f(t)=?$

$$F(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s-2)}+\frac{B_1}{s}$$

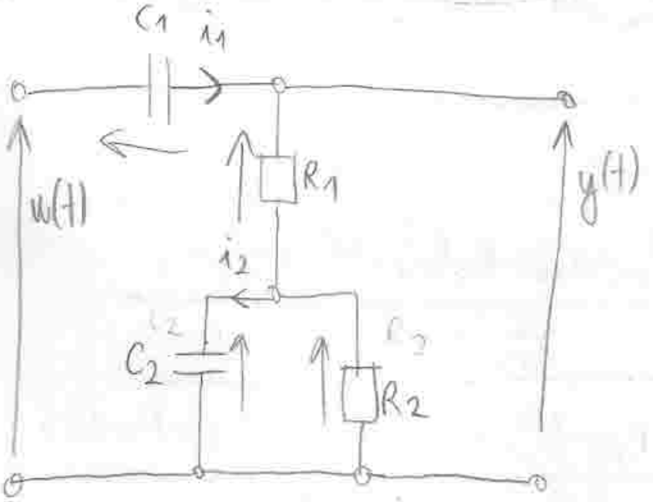
$$f(t)=A_1\cdot e^{-t}+A_2\cdot e^{2\cdot t}+B\cdot 1(t)$$

$$A_1=\frac{(s^2+1)\cdot\cancel{(s+1)}}{s\cdot\cancel{(s+1)}(s-2)}\Bigg|_{s=-1}=\frac{(-1)^2+1}{-1\cdot(-1-2)}=\frac{2}{-1\cdot(-3)}=\frac{2}{3}$$

$$A_2=\frac{(s^2+1)\cdot\cancel{(s-2)}}{s\cdot(s+1)\cdot\cancel{(s-2)}}\Bigg|_{s=2}=\frac{2^2+1}{2\cdot(2+1)}=\frac{5}{6}$$

$$B=\frac{(s^2+1)\cdot\cancel{s}}{\cancel{s}\cdot(s+1)(s-2)}\Bigg|_{s=0}=\frac{0+1}{(0+1)(0-2)}=\frac{1}{-2}=-\frac{1}{2}$$

$$f(t)=\frac{2}{3}\cdot e^{-t}+\frac{5}{6}\cdot e^{2\cdot t}-\frac{1}{2}\cdot 1(t)$$



$\mathcal{L}[u(t)] = U(s)$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}[i_1(t)] = I_1(s)$

$\mathcal{L}[i_2(t)] = I_2(s)$

$\tau = R \cdot C \; [s]$

$\tau = \frac{L}{R} \; [s]$

$$G(s) = \frac{Y(s)}{U(s)}$$

$$\begin{cases} u(t) = \frac{1}{C_1} \cdot \int_0^t i_1(t) \cdot dt + R_1 \cdot i_1(t) + \frac{1}{C_2} \int_0^t i_2(t) \cdot dt \\ 0 = \frac{1}{C_2} \cdot \int_0^t i_2(t) \cdot dt - R_2 \left(i_1(t) - i_2(t) \right) \\ y(t) = R_1 \cdot i_1(t) + R_2 \cdot (i_1(t) - i_2(t)) \end{cases}$$

$$\begin{cases} U(s) = \frac{1}{C_1} \cdot \frac{I_1(s)}{s} + R_1 \cdot I_1(s) + \frac{1}{C_2} \cdot \frac{I_2(s)}{s} & (1) \\ 0 = \frac{1}{C_2} \cdot \frac{I_2(s)}{s} - R_2 \cdot \left(I_1(s) - I_2(s) \right) & (2) \\ Y(s) = R_1 \cdot I_1(s) + R_2 \left(I_1(s) - I_2(s) \right) & (3) \end{cases}$$

$$R_2 \cdot I_1(s) - R_2 \cdot I_2(s) = \frac{1}{C_2 \cdot s} \cdot I_2(s)$$

$$R_2 \cdot I_1(s) = \frac{1}{C_2 \cdot s} \cdot I_2(s) + R_2 \cdot I_2(s)$$

$$R_2 \cdot I_1(s) = I_2(s) \cdot \left(\frac{1}{C_2 \cdot s} + R_2 \right)$$

$$I_2(s) = \frac{R_2 \cdot I_1(s)}{\frac{1}{C_2 \cdot s} + R_2}$$

$$U(s) = \frac{1}{C_1} \cdot \frac{I_1(s)}{s} + R_1 \cdot I(s) + \frac{1}{C_2 \cdot s} \cdot \frac{R_2 \cdot I_1(s)}{\frac{1}{C_1 \cdot s} + R_2}$$

$$Y(s) = R_1 \cdot I_1(s) + R_2 \cdot I_1(s) - R_2 \cdot \frac{R_2 \cdot I_1(s)}{\frac{1}{C_2 \cdot s} + R_2}$$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{\cancel{I_1(s)}\cdot\left[R_1+R_2-\frac{R_2^2}{\frac{1}{C_2\cdot s}+R_2}\right]}{\cancel{I_1(s)}\cdot\left[\frac{1}{C_1\cdot s}+R_1+\frac{R_2}{\frac{1}{C_2\cdot s}+R_2}\right]}$$

$$G(s)=\frac{R_1+R_2-\frac{R_2^2}{\frac{1}{C_2\cdot s}+R_2}}{R_1+\frac{1}{C_1\cdot s}+\frac{R_2}{\frac{1}{C_2\cdot s}+R_2}}=\frac{\frac{R_1\left(\frac{1}{C_2\cdot s}+R_2\right)+R_2\left(\frac{1}{C_2\cdot s}+R_2\right)-R_2^2}{\frac{1}{C_2\cdot s}+R_2}}{\frac{R_1\cdot C_1\cdot s\cdot\left(\frac{1}{C_2\cdot s}+R_2\right)+\left(\frac{1}{C_2\cdot s}+R_2\right)+R_2\cdot C_1\cdot s}{C_1\cdot s\cdot\left(\frac{1}{C_2\cdot s}+R_2\right)}}$$

$$G(s)=$$

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$2\ddot{x}+3\dot{x}+x=4 \qquad x(0)=1 \quad \dot{x}(0)=1$

$$2\frac{d^2x}{dt^2}+3\frac{dx}{dt}+x=4 \qquad \mathcal{L}[x(t)]=X(s)$$

$$\mathcal{L}\left[2\cdot\frac{d^2x}{dt^2}\right]+\mathcal{L}\left[3\frac{dx}{dt}\right]+\mathcal{L}[x(t)]=4$$

$$2\cdot\mathcal{L}\left[\frac{d^2x}{dt^2}\right]+3\mathcal{L}\left[\frac{dx}{dt}\right]+\mathcal{L}[x(t)]=4$$

$$2\cdot(s^2\cdot X(s)-s-1)+3(s\cdot X(s)-1)+X(s)=4$$

$$2\cdot s^2\cdot X(s)-2s-2+3\cdot s\cdot X(s)-3+X(s)=4$$

$$2s^2\cdot X(s)+3sX(s)+X(s)=4+2s+2+3=$$

$$X(s)\cdot(2s^2+3s+1)=9+2s$$

$$X(s)=\frac{9+2s}{2s^2+3s+1}$$

$$\Delta=b^2-4ac$$
$$\Delta=\left(\frac{3}{2}\right)^2-4\cdot1\cdot\frac{1}{2}$$
$$\Delta=\frac{9}{4}-2$$
$$\Delta=\frac{1}{4}$$

$$X(s)=\frac{s+4\frac{1}{2}}{s^2+1\frac{1}{2}s+\frac{1}{2}}$$

$$x_1=\frac{-b-\sqrt{\Delta}}{2a}=\frac{-\frac{3}{2}-\sqrt{\frac{1}{4}}}{2\cdot1}=\frac{-2}{2}=-1$$

$$x_2=\frac{-b+\sqrt{\Delta}}{2a}=\frac{-\frac{3}{2}+\sqrt{\frac{1}{4}}}{2\cdot1}=\frac{-1}{2}=-\frac{1}{2}$$

$$X(s)=\frac{s+\frac{9}{2}}{(s+1)(s+\frac{1}{2})}$$

$$X(s)=\frac{A_1}{(s+1)}+\frac{A_2}{(s+\frac{1}{2})}$$

$$x(t)=A_1\cdot e^{-t}+A_2\cdot e^{-\frac{1}{2}\cdot t}$$

Obliczamy stałe A_1 i A_2 z wzorów Heavisidea

$$A_1=\frac{s+\frac{9}{2}}{\cancel{(s+1)}(s+\frac{1}{2})}\cdot\cancel{(s+1)}\Bigg|_{s=-1}=\frac{-1+\frac{9}{2}}{-1+\frac{1}{2}}=\frac{\frac{7}{2}}{-\frac{1}{2}}=-7$$

$$A_2=\frac{s+\frac{9}{2}}{(s+1)\cancel{(s+\frac{1}{2})}}\cdot\cancel{(s+\frac{1}{2})}\Bigg|_{s=-\frac{1}{2}}=\frac{-\frac{1}{2}+\frac{9}{2}}{-\frac{1}{2}+1}=\frac{\frac{8}{2}}{\frac{1}{2}}=8$$

$$x(t)=-7\cdot e^{-t}+8\cdot e^{-\frac{1}{2}\cdot t}$$

$$F(s)=\frac{s^2+2s+1}{(s+3)^3\cdot s^2}$$

$$F(s)=\frac{A_1}{(s+3)}+\frac{A_2}{(s+3)^2}+\frac{A_3}{(s+3)^3}+\frac{B_1}{s}+\frac{B_2}{s^2}$$

$$f(t)=A_1\cdot e^{-3t}+A_2\cdot t\cdot e^{-3t}+A_3\cdot\frac{1}{2}\cdot t^2\cdot e^{-3t}+B_1\cdot 1(t)+B_2\cdot t$$

$$A_3=\frac{(s^2+2s+1)\cdot\cancel{(s+3)^3}}{\cancel{(s+3)^3}\cdot s^2}\Bigg|_{s=-3}=\frac{(-3)^2+2\cdot(-3)+1}{(-3)^2}=\frac{9-6+1}{9}=\frac{4}{9}$$

$$A_2=\frac{1}{1!}\cdot\frac{d}{ds}\left(\frac{s^2+2s+1}{s^2}\right)\Bigg|_{s=-3}=(2s+2)\cdot s^{-2}-2\cdot s^{-3}\cdot(s^2+2s+1)\Bigg|_{s=-3}=\frac{2\cdot(-3)+2}{(-3)^2}-2\cdot\frac{(-3)^2+2(-3)+1}{(-3)^3}=$$

$$=-\frac{4}{9}-2\cdot\frac{9-6+1}{(-27)}=-\frac{4}{9}+\frac{2\cdot 4}{27}=-\frac{12}{27}+\frac{8}{27}=-\frac{4}{27}$$

$$A_1=\frac{1}{2!}\frac{d}{ds}\left(\left[(2s+2)\cdot s^{-2}\right]-2\cdot s^{-3}\cdot(s^2+2s+1)\right)\Bigg|_{s=-3}=\left[2\cdot s^{-2}-2\cdot s^{-3}\cdot(2s+2)\right]-2\left[(2s+2)\cdot s^{-3}-3\cdot s^{-4}\cdot(s^2+2s+1)\right]\Bigg|_{s=-3}=$$

$$=\frac{1}{2}\cdot\left[\left[\frac{2}{(-3)^2}-\frac{2}{(-3)^3}\cdot(2\cdot(-3)+2)\right]-2\cdot\left[(2\cdot(-3)+2)\cdot\frac{1}{(-3)^3}-3\cdot\frac{1}{(-3)^4}\cdot((-3)^2+2\cdot(-3)+1\right]\right]=$$

$$=\frac{1}{2}\cdot\left[\frac{2}{9}+\frac{2\cdot(-4)}{27}-2\cdot\left(\frac{4}{27}-\frac{3\cdot(9-6+1)}{81}\right)\right]=\frac{1}{2}\cdot\left[\frac{2}{9}-\frac{8}{27}-\frac{8}{27}+\frac{24}{81}\right]=$$

$$=\frac{1}{2}\cdot\left[\frac{6}{27}-\frac{8}{27}-\cancel{\frac{8}{27}}+\cancel{\frac{8}{27}}\right]=\frac{1}{2}\cdot\left(-\frac{2}{27}\right)=-\frac{1}{27}$$

$$B_2=\frac{(s^2+2s+1)\cdot\cancel{s^2}}{(s+3)^3\cdot\cancel{s^2}}\Bigg|_{s=0}=\frac{0^2+2\cdot 0+1}{(0+3)^3}=\frac{1}{27}$$

$$B_1=\frac{1}{1!}\cdot\frac{d}{ds}\left[(s^2+2s+1)\cdot(s+3)^{-3}\right]\Bigg|_{s=0}=\left[(2s+2)\cdot(s+3)^{-3}-3\cdot(s+3)^{-4}\cdot 1\cdot(s^2+2s+1)\right]\Bigg|_{s=0}=$$

$$=\frac{2\cdot 0+2}{(0+3)^3}-\frac{3\cdot 1\cdot(0^2+2\cdot 0+1)}{(0+3)^4}=\frac{2}{27}-\frac{3}{81}=\frac{2}{27}-\frac{1}{27}=\frac{1}{27}$$

$$f(t)=-\frac{1}{27}\cdot e^{-3\cdot t}-\frac{4}{27}\cdot t\cdot e^{-3\cdot t}+\frac{2}{9}\cdot t^2\cdot e^{-3\cdot t}+\frac{1}{27}\cdot 1(t)+\frac{1}{27}\cdot t$$

$$G(s)=\frac{(s+1)^2}{s(0{,}1s+1)(100s+1)}$$

$z=a+bi$

$|z|=\sqrt{a^2+b^2}$

$\log_a b=c \quad a^c=b$

$$G(j\omega)=\frac{(j\omega+1)^2}{j\omega(0{,}1j\omega+1)(100j\omega+1)}$$

$$|G(j\omega)|=A(\omega)=\frac{\sqrt{\omega^2+1}^{\,2}}{\omega\left(\sqrt{(0{,}1\omega)^2+1}\right)\left(\sqrt{(100\omega)^2+1}\right)}$$

$L(\omega)=20\log A(\omega)$

$$L(\omega)=\underbrace{\overset{2\cdot 20}{40}\log\sqrt{\omega^2+1}}_{L_1}-\underbrace{20\log\omega}_{L_2}-\underbrace{20\log\sqrt{(0{,}1\omega)^2+1}}_{L_3}-\underbrace{20\log\sqrt{(100\omega)^2+1}}_{L_4}$$

$$\varphi(\omega)=\operatorname{arctg}\frac{Im}{Re}$$

$$\varphi(\omega)=\underbrace{2\operatorname{arctg}\frac{\omega}{1}}_{\varphi_1}-\underbrace{\operatorname{arctg}\frac{\omega}{0}}_{\varphi_2}-\underbrace{\operatorname{arctg}\frac{\omega\cdot 0{,}1}{1}}_{\varphi_3}-\underbrace{\operatorname{arctg}\frac{100\omega}{1}}_{\varphi_4}$$

$$\operatorname{arctg}\frac{\omega}{0}=\operatorname{arctg}\infty=90^0$$

$s^2+2s-3=0$

$$G(s)=\frac{(2s+1)^2}{s(s^2+2s-3)}$$

$\Delta=b^2-4a\cdot c$

$\Delta=2^2-4\cdot 1\cdot(-3)=4+12=16$

$x_1=\frac{-b-\sqrt{\Delta}}{2a}$

$x_1=\frac{-2-\sqrt{16}}{2\cdot 1}=\frac{-2-4}{2}=-3$

$x_2=-\frac{b+\sqrt{\Delta}}{2a}=\frac{-2+4}{2}=1$

$$G(s)=\frac{(2s+1)^2}{s(s+3)(s-1)}$$

$|z|=\sqrt{Re^2+Im^2}$

$$G(s)=\frac{(2s+1)^2}{3s(\frac{1}{3}s+1)(s-1)}$$

$$G(j\omega)=\frac{(2j\omega+1)^2}{3j\omega(\frac{1}{3}j\omega+1)(j\omega-1)}$$

$$A(\omega)=\frac{\sqrt{(2\omega)^2+1}^{\,2}}{\sqrt{(3\omega)^2}\cdot\sqrt{(\frac{1}{3}\omega)^2+1}\cdot\sqrt{\omega^2+1}}$$

$$L(\omega)=20\log A(\omega)=\underbrace{2\cdot 20\log\sqrt{(2\omega)^2+1}}_{L_1}-\underbrace{20\log\sqrt{(3\omega)^2}}_{L_2}-\underbrace{20\log\sqrt{(\frac{1}{3}\omega)^2+1}}_{L_3}-\underbrace{20\log\sqrt{\omega^2+1}}_{L_4}$$

$$\varphi(\omega)=\operatorname{arctg}\left(\frac{Im}{Re}\right)=2\cdot\operatorname{arctg}2\omega-\operatorname{arctg}\frac{3\omega}{0}-\operatorname{arctg}\frac{1}{3}\omega-\operatorname{arctg}\omega$$

$\hookrightarrow \operatorname{arctg}\infty=90^\circ$

$$\varphi(\omega)=\underbrace{2\operatorname{arctg}2\omega}_{\varphi_1}-90^0-\underbrace{\operatorname{arctg}\frac{1}{3}\omega}_{\varphi_2}-\underbrace{\operatorname{arctg}\omega}_{\varphi_3}$$

$$G(s)=\frac{(s+1)^2}{s(0{,}1s+1)(100s+1)}$$

$$\{|z| \quad z=a+bi$$
$$|z|=\sqrt{a^2+b^2}$$

$$G(j\omega)=\frac{(j\omega+1)^2}{j\omega(0{,}1j\omega+1)(100j\omega+1)}$$

$$G(j\omega)=\frac{(j\omega+1)(j\omega+1)}{j\omega(0{,}1j\omega+1)(100j\omega+1)}$$

$$|G(j\omega)|=\frac{\left(\sqrt{\omega^2+1}\right)^2}{\omega\left(\sqrt{0{,}01\omega^2+1}\right)\sqrt{100\omega^2+1}}=A(\omega)$$

$$L(\omega)=20\log A(\omega)$$

$$L(\omega)=\underbrace{2\cdot 20\log\sqrt{\omega^2+1}}_{L_1}-\underbrace{20\log\omega}_{L_2}-\underbrace{20\log\sqrt{0{,}01\omega^2+1}}_{L_3}-\underbrace{20\log\sqrt{100\omega^2+1}}_{L_4}$$

$$\varphi(\omega)=\arg G(j\omega)=\operatorname{arctg}\frac{\operatorname{Im}G(j\omega)}{\operatorname{Re}G(j\omega)}$$

$$\varphi(\omega)=\underbrace{2\operatorname{arctg}\frac{\omega}{1}}_{\varphi_1}-\underbrace{\operatorname{arctg}\frac{\omega}{0}}_{\varphi_2}-\underbrace{\operatorname{arctg}\frac{0{,}1\omega}{1}}_{\varphi_3}-\underbrace{\operatorname{arctg}\frac{100\omega}{1}}_{\varphi_4}$$

$$Y(s)=\frac{8\omega}{(s+1)(s^2+\omega^2)}=8\cdot\frac{\omega}{(s^3+\omega^2 s+s^2+\omega^2)}=8\frac{\omega}{s^3+s(\omega^2+s)+\omega^2}=\frac{8}{s(s^2+\omega^2+s)+\omega^2}$$

$$Y(s)=\frac{8\omega}{(s+1)(s+\omega)(s+\omega)-2\omega s}=\frac{8\omega}{(s+1)[(s+\omega)^2-2\omega s]}=$$

$$=\frac{8\omega}{(s+1)(s^2+2\omega s+\omega^2-2\omega s)}=\frac{8\omega}{s^3+2\omega s^2+\omega^2 s-2\omega s^2+s^2+2\omega s+\omega^2-2\omega s}=$$

$=$

$$\frac{8\omega}{(-2\omega^2-2j\omega)}\quad\frac{4\omega}{-\omega^2-j\omega}=\frac{4\omega(-\omega^2+j\omega)}{(-\omega^2-j\omega)(-\omega^2+j\omega)}=\frac{-4\omega^3+4j\omega^2}{\omega^4-\omega^3 j\omega+\omega^3 j\omega+\omega^2}=$$

$$=\frac{-4\omega^3+4\omega^2\cdot j}{\omega^4+\omega^2}=\frac{\omega^2(-4\omega+4j)}{\omega^2\cdot(\omega^2+1)}=\frac{-4\omega+4j}{\omega^2+1}$$

$$\frac{-4\omega-4j}{\omega^2+1}$$

$$\frac{8\omega}{1+\omega^2}\cdot\frac{1}{(s+1)}-\frac{4(\omega-j)}{1+\omega^2}\cdot\frac{1}{(s+j\omega)}-\frac{4(\omega+j)}{1+\omega^2}\frac{1}{(s-j\omega)}$$

$$\frac{8\omega}{1+\omega^2}\cdot\frac{(s+j\omega)(s-j\omega)}{(s+1)(s+j\omega)(s-j\omega)}-\frac{4(\omega-j)}{1+\omega^2}\cdot\frac{(s-j\omega)(s+1)}{(s+j\omega)(s-j\omega)(s+1)}-\frac{4(\omega+j)}{1+\omega^2}\frac{(s+j\omega)(s+1)}{(s+j\omega)(s-j\omega)(s+1)}=$$

$$=\frac{8\omega\cdot(s+j\omega)(s-j\omega)-4(\omega-j)(s-j\omega)(s+1)-4(\omega+j)(s+j\omega)(s+1)}{(1+\omega^2)(s+1)(s+j\omega)(s-j\omega)\ (*)}=$$

$$=\frac{8\omega(s^2+\omega^2)-4(\omega-j)(s^2+s-sj\omega-j\omega)-4(\omega+j)(s^2+s+sj\omega+j\omega)}{(*)}=$$

$$=8\omega^3+8s^2\omega-4[(\omega s^2+\omega s-s\cancel{j}\omega^2-\cancel{j\omega^2}-\cancel{j}s^2-\cancel{j}s+\cancel{s\omega}+\cancel{\omega})+(\omega s^2+\omega s+\cancel{\omega^2 sj}+\cancel{j\omega^2}+\cancel{j}s^2+\cancel{j}s-\cancel{s\omega}-\cancel{\omega})]$$

$$=\frac{8\omega^3+8s^2\omega-4(2\omega s^2+2\omega s)}{(*)}=\frac{8\omega^3+\cancel{8s^2\omega}-\cancel{8\omega s^2}-8\omega s}{(*)}=\frac{8\omega^3-8\omega s}{\omega^2+1}=$$

$$y(t)=\frac{8\omega}{1+\omega^2}\cdot e^{-1t}+\frac{4\omega}{(-\omega^2-j\omega)}\cdot e^{-j\omega t}+\frac{4\omega}{(-\omega^2+j\omega)}\cdot e^{j\omega t}$$

$$Y(s)=\frac{A_1}{s+1}+\frac{A_2}{s+j\omega}+\frac{A_3}{s-j\omega}$$

$$\frac{8\omega}{1+\omega^2}\cdot(s+j\omega)(s-j\omega)+\frac{4\omega}{(-\omega^2-j\omega)}\cdot(s+1)(s+j\omega)+\frac{4\omega}{(-\omega^2+j\omega)}(s+1)(s-j\omega)=$$

$$=\frac{8\omega}{1+\omega^2}(s^2+\omega^2)+\frac{4\omega}{(-\omega^2-j\omega)}(s^2+sj\omega+s+j\omega)+\frac{4\omega}{(-\omega^2+j\omega)}(s^2-sj\omega+s-j\omega)=$$

$$=\frac{8\omega s^2+8\omega^3}{1+\omega^2}+\frac{4\omega s^2+4\omega^2 sj+4\omega s+4j\omega^2}{(-\omega^2-j\omega)}+\frac{4\omega s^2-4\omega^2 sj+4\omega s-4j\omega^2}{(-\omega^2+j\omega)}=$$

$$=\frac{\omega(8s^2+8\omega^2)}{1+\omega^2}+\frac{\cancel{\omega}(4s^2+4\omega sj+4s+4j\omega)}{\cancel{\omega}(-\omega-j)}+\frac{\cancel{\omega}(4s^2-4\omega sj+4s-4j\omega)}{\cancel{\omega}(-\omega+j)}=$$

$$=\frac{8\omega s^2+8\omega^2}{1+\omega^2}+\frac{(4s^2+4\omega sj+4s+4j\omega)(-\omega+j)}{\omega^2\cancel{-j\omega}\cancel{+j\omega}+1}+\frac{(4s^2-4\omega sj+4s-4j\omega)(-\omega-j)}{\omega^2\cancel{+j\omega}\cancel{-j\omega}+1}=$$

$$=\frac{8\omega s^2+8\omega^2}{1+\omega^2}+\frac{(-4\omega s^2-\cancel{4\omega^2 js}-4\omega s\cancel{+4j\omega^2}+4s^2j-4\omega s\cancel{+4sj}-4\omega)}{1+\omega^2}+$$

$$+\frac{-4\omega s^2+\cancel{4\omega^2 sj}-4\omega s\cancel{+4j\omega^2}-4s^2j-4\omega s-\cancel{4sj}-4\omega}{1+\omega^2}=$$

$$=\frac{8\omega s^2+8\omega^2-4\omega s^2-4\omega s-4\omega s-4\omega-4\omega s^2-4\omega s-4\omega s-4\omega}{1+\omega^2}=$$

$$=\frac{\cancel{8\omega s^2}+8\omega^2-\cancel{8\omega s^2}-16\omega s-8\omega}{(1+\omega^2)}=\frac{8\omega^2-16\omega s-8\omega}{1+\omega^2}$$

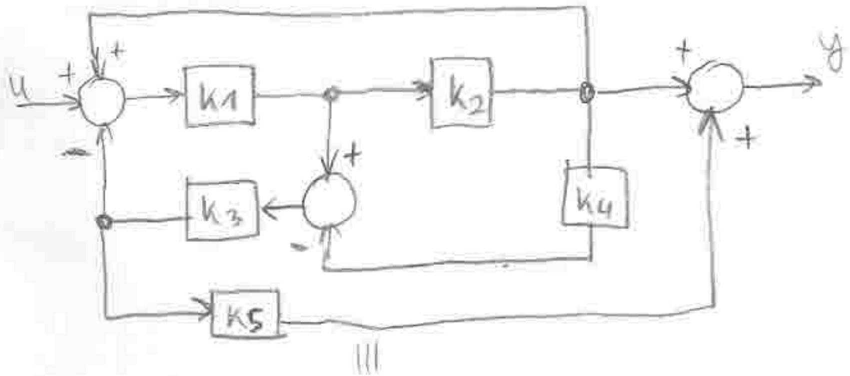
$A_1s^2+A_1\omega^2+A_2s^2+A_2sj\omega+A_2s+A_2j\omega+A_3s^2-A_3sj\omega+A_3s-A_3j\omega=$

$A_1+A_2+A_3=0$

$A_2-A_3=0$

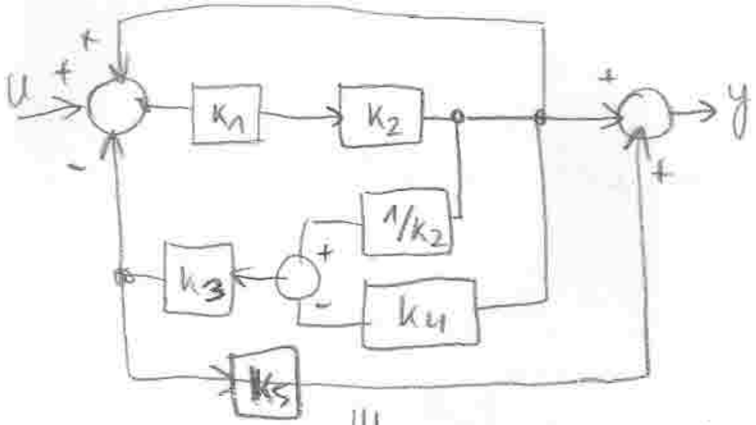
$A_2-A_3=0$

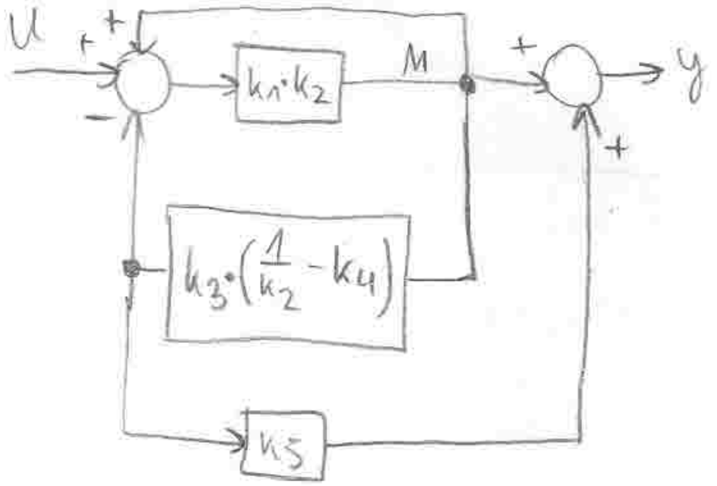
$A_2-A_3=0$

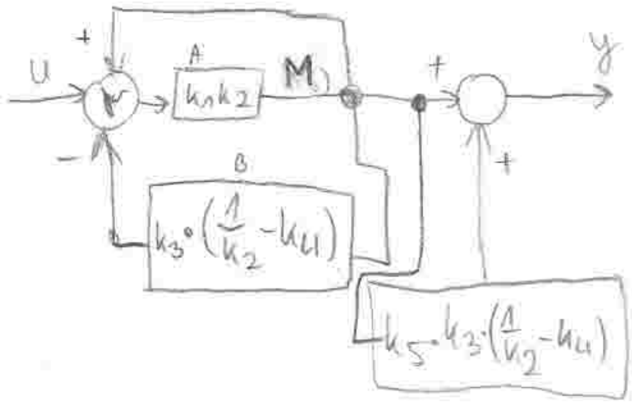


$A_{CL} = \frac{A}{1+AB}$ ujemne

$A_{CL} = \frac{A}{1-AB}$ dodatnie







$$M = \gamma \cdot (k_1 \cdot k_2)$$

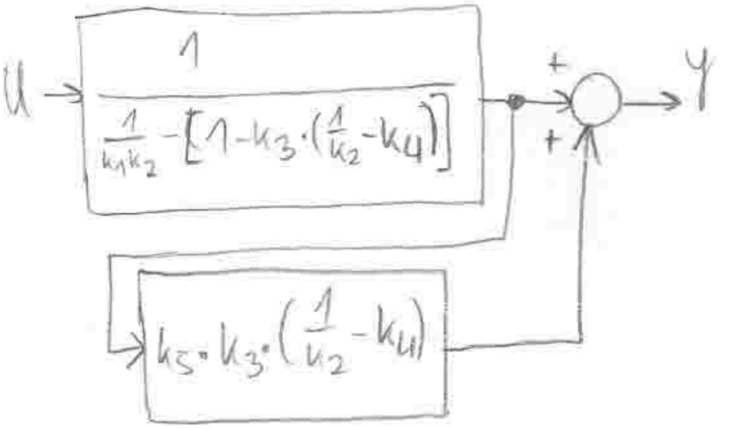
$$\gamma = U + M - k_3 \cdot \left(\frac{1}{k_2} - k_4\right) \cdot M$$

$$\frac{M}{k_1 k_2} = U + M\left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]$$

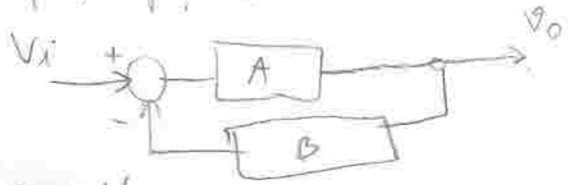
$$\frac{M}{k_1 \cdot k_2} - M\left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right] = U$$

$$M \cdot \left[\frac{1}{k_1 \cdot k_2} - \left(1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right)\right] = U$$

$$\frac{M}{U} = \frac{1}{\frac{1}{k_1 k_2} - \left(1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right)} =$$



ujemne sprzężenie zwrotne :



$$V_0 = A \cdot V_x$$

$$V_x = V_i - B \cdot V_0$$

$$V_0 = A \cdot [V_i - B \cdot V_0] = A \cdot V_i - AB \cdot V_0$$

$$V_0 + AB \cdot V_0 = A \cdot V_i$$

$$V_0(1 + A \cdot B) = A \cdot V_i$$

$$\frac{V_0}{V_i} = \frac{A}{1+AB} = A_{CL}$$

A_{CL} - amplification with closed loop

$$\frac{1}{\frac{1}{k_1 \cdot k_2} - \left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]} = \frac{1}{\frac{1 - k_1 \cdot k_2 \cdot \left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]}{k_1 \cdot k_2}} = \frac{k_1 \cdot k_2}{1 - k_1 \cdot k_2 \cdot \left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]}$$

$$U \rightarrow \boxed{\frac{k_1 \cdot k_2}{1 - k_1 \cdot k_2 \left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]} + \frac{k_1 \cdot k_2 \cdot k_5 \cdot k_3 \cdot \left(\frac{1}{k_2} - k_4\right)}{1 - k_1 \cdot k_2 \left[1 - k_3 \cdot \left(\frac{1}{k_2} - k_4\right)\right]}} \rightarrow y$$

$$U \rightarrow \boxed{\frac{k_1 \cdot k_2 + k_1 k_2 \cdot k_3 \cdot k_5 \left(\frac{1}{k_2} - k_4\right)}{1 - k_1 k_2 \left[1 - k_3 \left(\frac{1}{k_2} - k_4\right)\right]}} \rightarrow y$$