Mechanika klasyczna $\vec{F} = m\cdot\vec{a}$ $\quad \vec{F} = \frac{d\vec{p}}{dt}$ $\quad \vec{F} = \frac{d(m\cdot\vec{v})}{dt}$ $\quad \vec{p} = m\cdot\vec{v}$

Szczególna teoria względności Einsteina

$$m(v) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \mapsto \vec{F} = \frac{dp}{dt} \qquad p = \frac{m_0\cdot v}{\sqrt{1-\frac{v^2}{c^2}}}$$

Energia kinetyczna $W = \int \vec{F}\circ d\vec{s}$ $\quad E_k = \int F\cdot ds$

$$E_k = \int F\cdot ds = \int \frac{dp}{dt}\cdot ds \qquad \left\{ \begin{array}{l} v = \frac{ds}{dt} \\ ds = v\cdot dt \end{array} \right\}$$

$$E_k = \int \frac{dp}{dt}\cdot v\cdot dt = \int dp\cdot v$$

całkując przez części (w granicach 0 i v_*)

Uwzględniając $\left\{ \begin{array}{l} \frac{1}{2}\cdot\frac{d(v^2)}{dv} = v \\ \frac{1}{2}\cdot d(v^2) = v\cdot dv \end{array} \right\}$

$$E_k = [p\cdot v]_0^{v_*} - \int_0^{v_*} p\cdot dv = [m\cdot v\cdot v]_0^{v_*} - \int_0^{v_*} \frac{m_0\cdot v}{\sqrt{1-\frac{v^2}{c^2}}}\cdot dv$$

$$E_k = m\cdot v^2 - \int_0^{v_*} \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{1}{2}\cdot d(v^2) = m\cdot v^2\cdot\frac{c^2}{c^2} - \frac{m_0}{2}\cdot\int_0^{v_*}\frac{d(v^2)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \left\{\frac{c^2}{c^2} = 1\right\}$$

Podstawienie $\left| \begin{array}{l} v^2 \to x \\ 1 - \frac{x}{c^2} = t \\ -\frac{1}{c^2}\cdot dx = dt \\ dx = -c^2\cdot dt \end{array} \right|$

$$E_k = m\cdot c^2\cdot\left[\frac{v^2}{c^2}\right] - \frac{m_0}{2}\cdot\int_0^{v_*}\frac{d(v^2)}{\sqrt{1-\frac{v^2}{c^2}}}$$

Rozwiązanie całki

$$\int t^{-\frac{1}{2}}\cdot(-1)\cdot c^2\cdot dt = -c^2\cdot 2\cdot t^{\frac{1}{2}} + \underset{\text{stała}}{\text{const}}$$

$$E_k = m\cdot c^2\cdot\left[\frac{v^2}{c^2}\right] - \frac{m_0}{2}\cdot(-1)\cdot c^2\cdot 2\cdot\left[\sqrt{1-\frac{v^2}{c^2}}\right]_0^{v_*}$$

$$E_k = m\cdot c^2\cdot\left[\frac{v^2}{c^2}\right] + m_0\cdot c^2\cdot\left[\sqrt{1-\frac{v^2}{c^2}} - 1\right]$$

$$E_k = m_0\cdot c^2\cdot\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{v^2}{c^2}\right) + m_0\cdot c^2\cdot\left[\sqrt{1-\frac{v^2}{c^2}} - 1\right]$$

$$E_k = m_0\cdot c^2\cdot\left[\frac{\frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\sqrt{1-\frac{v^2}{c^2}}\cdot\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right] = m_0\cdot c^2\cdot\left[\frac{\frac{v^2}{c^2} + 1 - \frac{v^2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]$$

$$E_k = m_0\cdot c^2\cdot\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right] \rightarrow E_k = m\cdot c^2 - m_0\cdot c^2$$

$\underline{E = m\cdot c^2}$ - energia całkowita cząstki relatywistycznej

$\underline{E_0 = m_0\cdot c^2}$ - energia spoczynkowa