$$\int x^3 \cdot \sqrt[3]{x^2+1} \cdot dx = \left\{ \begin{array}{l} x^2+1 = t \rightarrow x^2 = t-1 \\ 2x \cdot dx = dt \\ x \cdot dx = \frac{1}{2} \cdot dt \end{array} \right\}$$

$$x^2+1 = x^2+1^2$$

$$= \int x^2 \cdot x \cdot \sqrt[3]{x^2+1} \cdot dx =$$

$$= \int (t-1) \cdot \sqrt[3]{t} \cdot \frac{1}{2} \cdot dt = \frac{1}{2} \cdot \int \left(t \cdot t^{\frac{1}{3}} - 1 \cdot t^{\frac{1}{3}} \right) dt = \frac{1}{2} \cdot \left(\int t^{\frac{4}{3}} \cdot dt - \int t^{\frac{1}{3}} \cdot dt \right) =$$

$$= \frac{1}{2} \left(\left[\frac{3}{7} \cdot t^{\frac{7}{3}} + C_1 \right] - \left[\frac{3}{4} \cdot t^{\frac{4}{3}} + C_2 \right] \right) = \frac{3}{14} \cdot t^{\frac{7}{3}} - \frac{3}{8} \cdot t^{\frac{4}{3}} + C = \{ C_1 + C_2 = C \}$$

$$= \frac{3}{14} \cdot \left(x^2+1\right)^{\frac{7}{3}} - \frac{3}{8} \cdot \left(x^2+1\right)^{\frac{4}{3}} + C$$