Macierz odwrotna – rozwiązane zadanie

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$det\boldsymbol{A} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} =$$

$$= \frac{\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix}}{\begin{matrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{matrix}} = 1\cdot 2\cdot 1 + 3\cdot 3\cdot 3 + 2\cdot 2\cdot 1 - (3\cdot 2\cdot 2 + 1\cdot 3\cdot 1 + 1\cdot 2\cdot 3) =$$

$$= 2 + 27 + 4 - (12 + 3 + 6) = 33 - 21 = 12$$

$$\boldsymbol{D}_{ij} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

$$d_{11} = (-1)^{1+1}\cdot det\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = 2 - 3 = -1$$

$$d_{12} = (-1)^{1+2}\cdot det\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = (-1)\cdot(3-2) = -1$$

$$d_{13} = (-1)^{1+3}\cdot det\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} = 9 - 4 = 5$$

$$d_{21} = (-1)^{2+1}\cdot det\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} = (-1)\cdot(2-9) = 7$$

$$d_{22} = (-1)^{2+2}\cdot det\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = 1 - 6 = -5$$

$$d_{23} = (-1)^{2+3}\cdot det\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = (-1)\cdot(3-4) = 1$$

$$d_{31} = (-1)^{3+1} \cdot det \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix} = 2 - 6 = -4$$

$$d_{32} = (-1)^{3+2} \cdot det \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} = (-1) \cdot (1 - 9) = 8$$

$$d_{33} = (-1)^{3+3} \cdot det \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = 2 - 6 = -4$$

$$\boldsymbol{D}_{ij} = \begin{bmatrix} -1 & -1 & 5 \\ 7 & -5 & 1 \\ -4 & 8 & -4 \end{bmatrix}$$

$$\boldsymbol{D}_{ij}{}^{T} = \begin{bmatrix} -1 & 7 & -4 \\ -1 & -5 & 8 \\ 5 & 1 & -4 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \frac{1}{det\boldsymbol{A}} \cdot \boldsymbol{D}_{ij}{}^{T}$$

$$\boldsymbol{A}^{-1} = \frac{1}{12} \cdot \begin{bmatrix} -1 & 7 & -4 \\ -1 & -5 & 8 \\ 5 & 1 & -4 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \begin{bmatrix} -\frac{1}{12} & \frac{7}{12} & -\frac{4}{12} \\ -\frac{1}{12} & -\frac{5}{12} & \frac{8}{12} \\ \frac{5}{12} & \frac{1}{12} & -\frac{4}{12} \end{bmatrix}$$

Sprawdzenie

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{12} & \frac{7}{12} & -\frac{4}{12} \\ -\frac{1}{12} & -\frac{5}{12} & \frac{8}{12} \\ \frac{5}{12} & \frac{1}{12} & -\frac{4}{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boldsymbol{I}$$

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \boldsymbol{I}$$