http://www.mbmaster.pl/matematyka-zadania.html

Macierz odwrotna

$$\boldsymbol{A}^{-1} = \frac{1}{det\boldsymbol{A}} \cdot \left[\boldsymbol{D}_{ij}\right]^{T}$$

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \boldsymbol{I}$$

$$\boldsymbol{A} - macierz$$

$$\boldsymbol{A}^{-1} - macierz\ odwrotna$$

$$\boldsymbol{I} - macierz\ jednostkowa$$

$$\left[\boldsymbol{D}_{ij}\right]^{T} - transponowana\ macierz\ dopełnień\ algebraicznych$$

$$i - numer\ wiersza$$

$$j - numer\ kolumny$$

$$D_{ij} = (-1)^{i+j} \cdot det\boldsymbol{A}_{bez\_i\_j}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$det\boldsymbol{A} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2$$

$$D_{11} = (-1)^{1+1} \cdot det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (-1)^{2} \cdot 4 = 4$$

$$D_{12} = (-1)^{1+2} \cdot det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (-1)^{3} \cdot 3 = -3$$

$$D_{21} = (-1)^{2+1} \cdot det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (-1)^{3} \cdot 4 = -2$$

$$D_{22} = (-1)^{2+2} \cdot det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (-1)^{4} \cdot 4 = 1$$

$$\boldsymbol{D}_{ij} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

$$\boldsymbol{D}_{ij} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

$$[\boldsymbol{D}_{ij}]^T = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \frac{1}{det\boldsymbol{A}} \cdot [\boldsymbol{D}_{ij}]^T$$

$$\boldsymbol{A}^{-1} = \frac{1}{(-2)} \cdot \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

Sprawdzenie

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 \cdot (-2) + 2 \cdot \frac{3}{2} & 1 \cdot 1 + 2 \cdot \left(-\frac{1}{2}\right) \\ 3 \cdot (-2) + 4 \cdot \frac{3}{2} & 3 \cdot 1 + 4 \cdot \left(-\frac{1}{2}\right) \end{bmatrix} =$$

$$= \begin{bmatrix} -2 + \frac{6}{2} & 1 + (-1) \\ -6 + \frac{12}{2} & 3 + \left(-\frac{4}{2}\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \boldsymbol{I}$$

$$\boldsymbol{A} \cdot \boldsymbol{A}^{-1} = \boldsymbol{I}$$