$$\frac{1}{10}\cdot\frac{1}{1-\left(-\frac{x}{2}\right)}=\frac{1}{10}\cdot\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n$$

http://www.mbmaster.pl

$$\frac{2}{5}\cdot\frac{1}{1-(2x)}=\frac{2}{5}\cdot\sum_{n=0}^{\infty}(2x)^n$$

$$f(x)=\frac{1}{10}\cdot\sum_{n=0}^{\infty}\left(-\frac{x}{2}\right)^n+\frac{2}{5}\cdot\sum_{n=0}^{\infty}(2x)^n \quad \text{dla } \underline{|x|<1}\;!$$

$$y'\cdot\cos x-y\cdot\sin x=\sin^2 x$$

R. J.

$$y'\cdot\cos x-y\cdot\sin x=0$$

$$y'\cdot\cos x=y\cdot\sin x$$

$$\frac{dy}{dx}\cdot\cos x=y\cdot\sin x \quad /:y\;/\cdot dx\;/:\cos x$$

$$\frac{dy}{y}=\frac{\sin x}{\cos x}\cdot dx$$

$$\int\frac{dy}{y}=\int\frac{\sin x}{\cos x}\cdot dx=\left\{\begin{array}{l}\cos x=t\\ -\sin x\cdot dx=dt\\ dx=-\frac{dt}{\sin x}\end{array}\right\}$$

$$\int\frac{dy}{y}=\int\frac{\cancel{\sin x}}{t}\cdot(-1)\cdot\frac{dt}{\cancel{\sin x}}$$

$$\int\frac{dy}{y}=-\int\frac{dt}{t}$$

$$\ln|y|+C_a=-\ln|t|+C_b$$

$$\ln|y|+C_a=-\ln|\cos x|+C_b$$

$$\ln|y|=-\ln|\cos x|+\underbrace{C_b-C_a}_{=C_c}$$

$$\ln|y|=-\ln|\cos x|+C_c$$

$$C_c=\ln C_*$$

$$\ln|y|=-\ln|\cos x|+\ln C_*$$

$$\left\{-\ln|a|=\ln\left|\frac{1}{a}\right|;\quad \ln a+\ln b=\ln(a\cdot b)\right\}$$

$$\ln|y|=\ln\left|\frac{1}{\cos x}\right|+\ln C_*$$

$$\ln|y|=\ln\left|\frac{C_*}{\cos x}\right|$$

$$|y|=\left|\frac{C_*}{\cos x}\right|$$

$$y=\underbrace{\pm\frac{C_*}{\cos x}}_{=C}$$

$$y=\frac{C}{\cos x}$$

uzmienniamy stałą C

$$y=\frac{C(x)}{\cos x}$$

$$y'=\frac{C'(x)}{\cos x}+\frac{(-1)\cdot C(x)}{\cos^2 x}\cdot(-1)\cdot\sin x$$

$$y'=\frac{C'(x)}{\cos x}+\frac{C(x)\cdot\sin x}{\cos^2 x}$$

wstawiamy do równania niejednorodnego

$$\left(\frac{C'(x)}{\cos x}+\frac{C(x)\cdot\sin x}{\cos^2 x}\right)\cdot\cos x-\frac{C(x)}{\cos x}\cdot\sin x=\sin^2 x$$

$$\left(\frac{C'(x)}{\cos x} + \frac{C(x)\cdot\sin x}{\cos^2 x}\right)\cdot\cos x - \frac{C(x)}{\cos x}\cdot\sin x = \sin^2 x$$

$$C'(x) + \frac{\cancel{C(x)}\cdot\sin x}{\cos x} - \frac{\cancel{C(x)}\cdot\sin x}{\cancel{\cos x}} = \sin^2 x$$

$$C'(x) = \sin^2 x \qquad \left\{ \frac{dC(x)}{dx} = C'(x) \right\}$$

$$C(x) = \int \sin^2 x\cdot dx \qquad \left\{ \begin{array}{l} 1-\cos 2x = 2\cdot\sin^2 x \\ \sin^2 x = \dfrac{1-\cos 2x}{2} \end{array} \right\}$$

$$C(x) = \int \frac{1-\cos 2x}{2}\cdot dx$$

$$C(x) = \int\left(\frac{1}{2} - \frac{\cos 2x}{2}\right)\cdot dx = \underbrace{\int \frac{1}{2}\cdot dx}_{*_1} + \underbrace{\int -\frac{\cos 2x}{2}\cdot dx}_{*_2}$$

$$*_1 = \int \frac{1}{2}\cdot dx = \frac{1}{2}\cdot x + C$$

$$*_2 = \int -\frac{\cos 2x}{2}\cdot dx = -\frac{1}{2}\cdot\int \cos 2x\cdot dx \left\{ \begin{array}{l} 2x = t \\ 2\cdot dx = dt \\ dx = \frac{1}{2}\cdot dt \end{array} \right\} = -\frac{1}{2}\cdot\int \cos t\cdot\frac{1}{2}\cdot dt =$$

$$= -\frac{1}{2}\cdot\frac{1}{2}\cdot\int \cos t\cdot dt = -\frac{1}{4}\cdot\sin t + C = -\frac{1}{4}\cdot\sin 2x + C$$

$$C(x) = \frac{1}{2}\cdot x - \frac{1}{4}\cdot\sin 2x + C$$

Wstawiamy $C(x)$ do $y = \frac{C(x)}{\cos x}$

$$y = \frac{\left(\frac{1}{2}\cdot x - \frac{1}{4}\cdot\sin 2x + C\right)}{\cos x}$$

$$\boxed{y = \frac{1}{2}\cdot\frac{x}{\cos x} - \frac{1}{4}\cdot\frac{\sin 2x}{\cos x} + \frac{C}{\cos x}}$$

$$y'' + 2\cdot y' + y = e^{-x}$$

RJ

$$y'' + 2y' + y = 0$$

$r^2 + 2\cdot r + 1 = 0$ ← równanie charakt.

$\Delta = b^2 - 4\cdot a\cdot c = 2^2 - 4\cdot 1\cdot 1 = 4-4 = 0$

$r_{0_1} = \frac{-b}{2\cdot a} = \frac{-2}{2\cdot 1} = -1$

pierwiastek 2-krotny

ponieważ $\Delta = 0$

Rozwiązanie równania jednorodnego:

$$y = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x}$$

Przewidywane rozwiązanie szczególne

$$y_p = Ax^2 \cdot e^{-x}$$

$\{ x^2 \to$ ponieważ (-1) jest 2-krotnym $\sqrt{\ }$

$$y_p' = A \cdot 2 \cdot x \cdot e^{-x} + A \cdot x^2 \cdot e^{-x} \cdot (-1)$$

$$y_p'' = A \cdot 2 \cdot e^{-x} + (-1) \cdot A \cdot 2 \cdot x \cdot e^{-x} + (-1) \cdot A 2 x \cdot e^{-x} + (-1) \cdot A \cdot x^2 \cdot e^{-x} \cdot (-1)$$

$$y_p'' = 2 \cdot A \cdot e^{-x} - 2 \cdot A \cdot x \cdot e^{-x} - 2 \cdot A \cdot x \cdot e^{-x} + A \cdot x^2 \cdot e^{-x}$$

Podstawiamy y_p i y_p' i y_p'' do równania niejednorodnego

$$2 \cdot A \cdot e^{-x} \cancel{-4 A \cdot x} \cdot e^{-x} + A \cdot x^2 \cdot e^{-x} + 2 \cdot (A \cdot 2 \cdot x \cdot e^{-x} - \cancel{A \cdot x^2} \cdot e^{-x}) +$$

$$+ \cancel{A \cdot x^2} \cdot e^{-x} = e^{-x}$$

$$2 \cdot A \cdot e^{-x} = e^{-x} \quad / : e^{-x}$$

$$2A = 1 \to A = \frac{1}{2} \longrightarrow y_s = \frac{1}{2} \cdot x^2 \cdot e^{-x}$$

Rozwiązanie końcowe

$$\boxed{y = C_1 \cdot e^{-x} + C_2 \cdot x \cdot e^{-x} + \frac{1}{2} \cdot x^2 \cdot e^{-x}}$$