

$$\begin{cases} C_1'(x) \cdot y_1(x) + C_2'(x) \cdot y_2(x) = 0 \\ C_1'(x) \cdot y_1'(x) + C_2'(x) \cdot y_2'(x) = \frac{1}{a} \cdot y(x) \end{cases}$$

R.N.

$$y = \underbrace{y_2(x)}_{R.S.} + \underbrace{C_1 y_1(x) + C_2 y_2(x)}_{= R.O.J.}$$

$$W = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$$

Przykład

$$y'' + 4y = \frac{1}{\cos 2x}$$

$$\text{R.O.J. } y'' + 4y = 0 \rightarrow r^2 + 4 = 0 \quad r^2 = -4 \quad r = \sqrt{-4} \quad r = \pm 2i$$

$$y = C_1 \cdot e^{0x} \cdot \cos 2x + C_2 \cdot e^{0x} \cdot \sin 2x$$

Uziennianie stoly ch:

$$y = C_1(x) \cdot \cos 2x + C_2(x) \cdot \sin 2x$$

$$y' = C_1'(x) \cdot \underbrace{\cos 2x}_{y_1} + C_1(x) \cdot \underbrace{(-2) \cdot \sin 2x}_{y_1'} + C_2'(x) \cdot \underbrace{\sin 2x}_{y_2} + C_2(x) \cdot \underbrace{2 \cdot \cos 2x}_{y_2'}$$

$$\begin{cases} C_1'(x) \cdot \cos 2x + C_2'(x) \cdot \sin 2x = 0 \\ C_1'(x) \cdot (-2) \cdot \sin 2x + C_2'(x) \cdot 2 \cdot \cos 2x = \frac{1}{\cos 2x} \end{cases}$$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2 \cdot \sin 2x & 2 \cdot \cos 2x \end{vmatrix} = \cos 2x \cdot 2 \cdot \cos 2x - [-2 \cdot \sin 2x \cdot \sin 2x] =$$

$$= 2 \cdot \cos^2 2x + 2 \cdot \sin^2 2x = 2 \cdot [\underbrace{\cos^2 2x + \sin^2 2x}_{=1}] = 2$$

$$W_{C_1}(x) = \begin{vmatrix} 0 & \sin 2x \\ \frac{1}{\cos 2x} & 2 \cdot \cos 2x \end{vmatrix} = 0 - \frac{\sin 2x}{\cos 2x} = -\frac{\sin 2x}{\cos 2x}$$

$$C_1'(x) = \frac{W_{C_1}(x)}{W} = -\frac{\sin 2x}{\cos 2x} \cdot \frac{1}{2} = -\frac{\sin 2x}{2 \cdot \cos 2x}$$

$$W_{C_2}(x) = \begin{vmatrix} \cos 2x & 0 \\ -2 \cdot \sin 2x & \frac{1}{\cos 2x} \end{vmatrix} = \frac{\cos 2x}{\cos 2x} - 0 = 1$$

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$$C_2'(x) = \frac{W_{C_2}(x)}{W} = \frac{1}{2}$$

$$C_1(x) = \int \frac{\sin 2x}{2 \cdot \cos 2x} \cdot dx = \begin{cases} \cos 2x = t \\ -2 \cdot \sin 2x = dt \\ -\sin 2x = \frac{dt}{2} \end{cases} = \int \frac{dt}{2 \cdot 2 \cdot t} = \int \frac{dt}{4 \cdot t} = \frac{1}{4} \cdot \int \frac{dt}{t} = \frac{1}{4} \cdot \ln|t| + C_1 =$$

$$C_1(x) = \frac{1}{4} \cdot \ln|\cos 2x| + C_1$$

$$C_2(x) = \int \frac{1}{2} \cdot dx = \frac{1}{2} \cdot \int dx = \frac{1}{2} \cdot x + C_2$$

$$y = C_1(x) \cdot \cos 2x + C_2(x) \cdot \sin 2x \leftarrow \text{ustawiamy uzmiennione stale do R.O.}$$

$$y = \left(\frac{1}{4} \cdot \ln|\cos 2x| + C_1 \right) \cdot \cos 2x + \left(\frac{1}{2} x + C_2 \right) \cdot \sin 2x$$

$$y = \frac{1}{4} \cdot \cos 2x \cdot \ln|\cos 2x| + C_1 \cdot \cos 2x + \frac{1}{2} x \cdot \sin 2x + C_2 \cdot \sin 2x$$

$$y = \underbrace{\frac{1}{4} \cdot \cos 2x \cdot \ln|\cos 2x| + \frac{1}{2} x \cdot \sin 2x}_{\text{R.S.}} + \underbrace{C_1 \cdot \cos 2x + C_2 \cdot \sin 2x}_{\text{R.O.}}$$