Kryterium d'Alemberta

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jeżeli $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \;$ >1 - szereg rozbieżny; $=1$ - nie wiadomo; <1 szereg jest zbieżny

$\frac{3^n}{3^{n+1}} = 3^{n-(n+1)} = 3^{-1}$

1)

$$\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{3^n\cdot n!}$$

$2(n+1)-1 = 2n+1$

$$\lim_{n\to\infty} \frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)\cdot(2(n+1)-1)}{3^{n+1}\cdot(n+1)!}\cdot\frac{3^n\cdot n!}{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)} =$$

$$= \lim_{n\to\infty} \frac{(2n-1)\cdot(2n+1)}{(2n-1)}\cdot 3^{n-(n+1)}\cdot\frac{n!}{n!(n+1)} = \lim_{n\to\infty}\frac{2n+1}{3\cdot(n+1)} = \left\{ \frac{1/n}{1/n} \right.$$

$$= \lim_{n\to\infty} \frac{2+\frac{1}{n}\nearrow 0}{3+\frac{3}{n}\searrow 0} = \frac{2}{3} < 1$$

szereg $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{3^n\cdot n!}$ jest zbieżny

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$$\sum_{n=1}^{\infty} \frac{3^n}{2n!}$$

$$\lim_{n\to\infty} \frac{3^{n+1}}{2(n+1)!}\cdot\frac{2n!}{3^n} = \lim_{n\to\infty} \frac{3^{n+1-n}}{2(n!(n+1))}\cdot\frac{2n!}{1} = \lim_{n\to\infty}\frac{3}{n+1} = \lim_{n\to\infty}\frac{\frac{3}{n}\nearrow 0}{1+\frac{1}{n}\searrow 0} = 0<1$$

szereg $\sum_{n=1}^{\infty} \frac{3^n}{2n!} \to$ zbieżny

$$\sum_{n=1}^{\infty} \frac{2^n+3^n}{5^n}$$

$$\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{5^{n+1}}\cdot\frac{5^n}{2^n+3^n} = \lim_{n\to\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}\cdot\frac{1}{5} = \lim_{n\to\infty}\frac{1}{5}\cdot\frac{\frac{2^{n+1}}{3^{n+1}}\nearrow 0+\frac{3^{n+1}}{3^{n+1}}\nearrow 1}{\frac{2^n}{3^{n+1}}+\frac{3^n}{3^{n+1}}\searrow 0} = \frac{3}{5}<1$$

$= \lim_{n\to\infty} \frac{1}{5} \cancel{\frac{1}{\frac{1}{3}}} = \lim_{n\to}$

↓

http://www.mbmaster.pl

Kryterium Cauchy'ego $\lim\limits_{n\to\infty}\sqrt[n]{a_n}$

$\sqrt[n]{2}=1$

$\sqrt[n]{a}=1$

$$\sum_{n=1}^{\infty}\frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n}$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{\left(1+\frac{1}{n}\right)^{n^2}}{2^n}}=\lim_{n\to\infty}\frac{1}{2}\cdot\sqrt[n]{\left(1+\frac{1}{n}\right)^{n^2}}=\lim_{n\to\infty}\frac{1}{2}\sqrt[n]{\left(1+\frac{1}{n}\right)^{n}\cdot\left(1+\frac{1}{n}\right)^{n}}=$$

$$=\lim_{n\to\infty}\frac{1}{2}\cdot\sqrt[n]{e^{n}}=\frac{1}{2}e\approx\frac{2{,}718}{2}>1$$ szereg rozbieżny

$$\sum_{n=1}^{\infty}\frac{n\cdot 7^n}{2^{n+1}\cdot 3^n}$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{n\cdot 7^n}{2^{n+1}\cdot 3^n}}=\lim_{n\to\infty}\frac{\overset{1}{\sqrt[n]{n}}\cdot\overset{7}{\sqrt[n]{7^n}}}{\underset{}{\sqrt[n]{2^{n+1}}}\cdot\underset{3}{\sqrt[n]{3^n}}}=\frac{7}{2\cdot 3}>1$$ szereg rozbieżny

$$\left(2^{n+1}\right)^{\frac{1}{n}}=\sqrt[n]{2}=1$$

$$\sum_{n=1}^{\infty}\frac{2^n+3^n}{5^n}$$

Twierdzenie o trzech ciągach.

$$\lim_{n\to\infty}\sqrt[n]{\frac{2^n+3^n}{5^n}}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{2}{5}\right)^n+\left(\frac{3}{5}\right)^n}$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{3}{5}\right)^n}<\lim_{n\to\infty}\sqrt[n]{\left(\frac{2}{5}\right)^n+\left(\frac{3}{5}\right)^n}<\lim_{n\to\infty}\sqrt[n]{\left(\frac{3}{5}\right)^n+\left(\frac{3}{5}\right)^n}$$

$$\lim_{n\to\infty}\sqrt[n]{2\cdot\left(\frac{3}{5}\right)^n}=\frac{3}{5}<1$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{3}{5}\right)^n}=\frac{3}{5}<1$$

$$\frac{3}{5}<1$$

$\sqrt[n]{2}=1$

$\sqrt[n]{\left(\frac{3}{5}\right)^n}=\frac{3}{5}$

szereg $\sum\limits_{n=1}^{\infty}\frac{2^n+3^n}{5^n}$ jest zbieżny