13. 12. 2010r

$$\sum_{n=1}^{\infty} \frac{1}{(5n+1)(5n+4)} = \frac{1}{25n^2+20n+5n+4} = \frac{1}{25n^2+25n+4}$$

$\Delta = b^2 - 4ac$

$\Delta = 25^2 - 4\cdot 25\cdot 4$

$\Delta = 625 - 400 = 225$

$\sqrt{\Delta} = 15$

$$\cancel{n_{01} = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-25+15}{2\cdot 25} = \frac{-40}{50} = -\frac{4}{5}}$$

$$\cancel{n_{02} = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-25+15}{2\cdot 25} = \frac{10}{50} = \frac{1}{5}}$$

$$\cancel{\sum_{n=1}^{\infty} \frac{1}{(n-\frac{4}{5})(n+\frac{1}{5})}} =$$

$\frac{4}{5}\cdot\frac{5}{3} - \frac{1}{5}\cdot\frac{5}{3} = 1$

$\frac{20}{15} - \frac{5}{15} = 1$ o.k.

$$\sum_{n=1}^{\infty} = \frac{1}{5(n+\frac{1}{5})\,5\cdot(n+\frac{4}{5})} = \frac{1}{25}\cdot\sum_{n=1}^{\infty}\frac{1}{(n+\frac{1}{5})(n+\frac{4}{5})}$$

$B = -\frac{5}{3}$

$B = -A$

$$\frac{1}{(n+\frac{1}{5})(n+\frac{4}{5})} = \frac{A}{n+\frac{1}{5}} + \frac{B}{n+\frac{4}{5}}$$

$n^1 \;|\; A+B=0$

$n^0 \;|\; \frac{4}{5}A + \frac{1}{5}B = 1$

$\frac{4}{5}A - \frac{1}{5}A = 1$

$\frac{3}{5}A = 1$

$A = \frac{5}{3}$

$$1 = A\cdot(n+\tfrac{4}{5}) + B\cdot(n+\tfrac{1}{5}) = A\cdot n + \tfrac{4}{5}\cdot A + B\cdot n + \tfrac{1}{5}\cdot B$$

$$\frac{1}{25}\sum_{n=1}^{\infty} \frac{5}{3}\cdot\frac{1}{(n+\frac{1}{5})} - \frac{5}{3}\cdot\frac{1}{(n+\frac{4}{5})}$$

$$S_1 = \frac{1}{25}\cdot\left(\frac{5}{3}\cdot\frac{5}{6} - \frac{5}{3}\cdot\frac{5}{9}\right) = \frac{5}{75}\cdot\left(\frac{5}{6} - \frac{5}{9}\right)$$

$$S_2 = \frac{5}{75}\cdot\left(\frac{5}{6} - \frac{5}{9} + \frac{5}{11} - \frac{5}{14} + \right.$$

$$\sum_{n=1}^{\infty}\frac{1}{25n^2+25+4} \leq \sum_{n=1}^{\infty}\frac{n^2}{25n^2+25n+4} = \sum_{n=1}^{\infty}\frac{1}{25+\frac{25n}{n^2}+\frac{4}{n^2}} = \frac{1}{25} \quad \text{- majoranta}$$

$$\neq \frac{1}{25}$$

$a^c = b$

$\log_a b = c$

$$\sum_{n=1}^{\infty}\frac{1}{n\cdot\ln n} \qquad f(x) = \frac{1}{x\cdot\ln x}$$

$$f(x)\int_2^{\infty}\frac{1}{x\cdot\ln x}\cdot dx = \left\{\begin{array}{l}\ln x = t\\ \frac{1}{x}\cdot dx = dt\\ dx = x\cdot dt\end{array}\right\} = \int_2^{\infty}\frac{x\cdot dt}{x\cdot t} = \lim_{\beta\to\infty}\int_2^{\beta}\frac{dt}{t} = \lim_{\beta\to\infty}\ln|t|\Big|_0^{\beta} =$$

$$= \lim_{\beta\to\infty}\ln\Big|_{\ln 2}^{\ln\beta} = \lim_{\beta\to\infty}\Big(\ln(\ln|\beta|) - \ln(\ln|2|)\Big) = \infty$$

szereg $\sum_{n=2}^{\infty}\frac{1}{n\cdot\ln n}$ jest rozbieżny do ∞

10.12.2010r

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

$x^2+1=t$
$2x \cdot dx = dt$

$$f(x)=\int_1^{+\infty} \frac{1}{x^2+1}dx = \lim_{\beta\to+\infty} \int_1^{\beta} \frac{1 \cdot dx}{x^2+1} = \lim_{\beta\to\infty} \operatorname{arctg}\frac{x}{1}\Big|_1^{\beta} = \lim_{\beta\to 0} \operatorname{arctg}\beta - \operatorname{arctg} 1 =$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \frac{\pi}{4}$$

$$\sum_{n=0}^{\infty} \frac{1}{n^2+4n+8} = \sum_{n=0}^{\infty} \frac{1}{n^2+4n+4+4} = \sum_{n=0}^{\infty} \frac{1}{(n+2)^2+4}$$

$$f(x)=\int_0^{+\infty} \frac{1\,dx}{(x+2)^2+4} = \lim_{\beta\to\infty}\int_0^{+\infty}\frac{1\,dx}{(x+2)^2+4} = \lim_{\beta\to\infty}\frac{1}{4}\int_0^{+\infty}\frac{dx}{\frac{(x+2)^2}{4}+1} = \lim_{\beta\to\infty}\frac{1}{4}\cdot\int_0^{\infty}\frac{dx}{\left(\frac{x+2}{2}\right)^2+1} = \Bigg\{ \Bigg\} =$$

$$= \left\{ \begin{array}{l} \frac{x+2}{2}=t \\ \frac{1}{2} dx = dt \\ dx = 2\,dt \end{array} \right\} = \lim_{\beta\to\infty}\frac{1}{4}\cdot\int_0^{\infty}\frac{2 \cdot dt}{t^2+1} = \lim_{\beta\to\infty}\frac{2}{4}\int_0^{\infty}\frac{dt}{t^2+1} = \lim_{\beta\to\infty}\frac{1}{2}\int_0^{\infty}\frac{1}{t^2+1} = \lim_{\beta\to\infty}\frac{1}{2}\cdot\operatorname{arctg} t\Big|_0^{+\infty} =$$

$$= \lim_{\beta\to\infty}\frac{1}{2}\cdot\left(\operatorname{arctg}\frac{x+2}{2}\Big|_0\right) = \lim_{\beta\to\infty}\frac{1}{2}\left(\operatorname{arctg}\frac{\infty+2}{2} - \operatorname{arctg} 1\right) = \frac{1}{2}\cdot\left(\frac{\pi}{2}-\frac{\pi}{4}\right) = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$

Szereg $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+8n}$ jest zbieżny

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27.12.2010r

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$$

$a_1 = \frac{1}{2}$
$a_2 = \frac{1}{4}$

$$q = \frac{a_{n+1}}{a_n} = \frac{1}{4}\cdot\frac{2}{1} = \frac{1}{2}$$

$S_1 = \frac{1}{2}$
$S_2 = \frac{1}{2}+\frac{1}{4}$

$$\lim_{n\to\infty} S_n = \sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n = \frac{a_1}{1-q} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1/2}{1/2} = 1$$

$$\sum_{n=1}^{\infty}\frac{3^n}{(2n!)}$$

$$a.\quad \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim \frac{3^{n+1}}{(2(n+1))!}\cdot\frac{2n!}{3^n} = \lim_{n\to\infty}\frac{3^n\cdot 3}{(2\cdot n!\cdot(n+1))}\cdot\frac{\cancel{2n!}}{\cancel{3^n}} = \lim_{n\to\infty}\frac{3}{n+1} = 0$$