$\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$ Kryterium d'Alemberta: $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ {04.01.2011r}

$$\lim_{n\to\infty} \frac{3^{n+1}}{[2(n+1)]!} \cdot \frac{(2n)!}{3^n} = \lim_{n\to\infty} \frac{\cancel{3^n}\cdot 3}{[2n+2]!} \cdot \frac{2n!}{\cancel{3^n}} = \lim_{n\to\infty} \frac{3}{2!(n+1)!} \cdot \frac{2n!}{1} = \lim_{n\to\infty} \frac{3}{2n!\cdot(n+1)} \cdot 2n! =$$

$$= \lim_{n\to\infty} \frac{3}{n+1} \cdot \frac{1/n}{1/n} = \frac{\frac{3}{n}}{1+\frac{1}{n}} = 0 \rightarrow \text{szereg } \sum_{n=1}^{\infty} \frac{3^n}{(2n)!} \text{ jest zbieżny}$$

$\sum_{n=1}^{\infty} \frac{n\cdot 7^n}{2^{n+1}\cdot 3^n}$. Kryterium Cauchy'ego

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n\cdot 7^n}{2^{n+1}\cdot 3^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}\cdot\sqrt[n]{7^n}}{\sqrt[n]{2^n}\cdot 2\cdot\sqrt[n]{3^n}} = \lim_{n\to\infty} \frac{1\cdot 7}{2\cdot 1\cdot 3} = \frac{7}{6} > 1$$

szereg $\sum_{n=1}^{\infty} \frac{n\cdot 7^n}{2^{n+1}\cdot 3^n}$ jest rozbieżny

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{05.01.2010r}

$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

$$\int_1^{\infty} \frac{1}{x^2+1}\,dx = \lim_{\beta\to\infty} \int_1^{\beta} \frac{1}{x^2+1}\,dx = \lim_{\beta\to\infty} \operatorname{arctg} x \Big|_1^{\beta} = \lim_{\beta\to\infty} [\operatorname{arctg}\beta - \operatorname{arctg} 1] =$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \longrightarrow \text{szereg } \sum_{n=1}^{\infty} \frac{1}{n^2+1} \text{ jest zbieżny do } \frac{\pi}{4}$$

$\sum_{n=1}^{\infty} \frac{1}{n^2+4n+8}$

$$\int_1^{\infty} \frac{1}{x^2+4x+8}\,dx = \lim_{\beta\to\infty} \int_1^{\beta} \frac{1}{x^2+4x+8}\,dx = \lim_{\beta\to\infty} \int_1^{\beta} \frac{1}{(x+2)^2+4}\,dx = \lim_{\beta\to\infty} \int_1^{\beta} \frac{1}{4}\cdot\frac{dx}{\frac{(x+2)^2}{4}+1} =$$

$$= \lim_{\beta\to\infty} \frac{1}{4} \int_1^{\beta} \frac{dx}{\left(\frac{x+2}{2}\right)^2+1} = \left\{ \begin{array}{l} \frac{x+2}{2} = t \\ \frac{1}{2}\,dx = dt \\ dx = 2\,dt \end{array} \right\} = \frac{1}{4}\cdot \lim_{\beta\to\infty} \int_1^{\beta} \frac{2\,dt}{t^2+1} = \frac{1}{4}\cdot\lim_{\beta\to\infty} 2\cdot \int_1^{\beta} \frac{dt}{t^2+1} =$$

$$= \frac{2}{4}\cdot \lim_{\beta\to\infty} \operatorname{arctg} t \Big|_1^{\beta} = \frac{1}{2}\cdot \lim_{\beta\to\infty} \operatorname{arctg} x \Big|_1^{\beta} = \frac{1}{2}\cdot\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{\pi}{8}$$

szereg $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+8}$ jest zbieżny do $\frac{\pi}{8}$

{06.01.2011r}

$\sum_{n=2}^{\infty} \frac{1}{n\cdot\ln n}$ $\quad f(x) = \frac{1}{x\cdot\ln x}$

$$\int_2^{\infty} \frac{dx}{x\cdot\ln x} = \left\{ \begin{array}{l} \ln x = t \\ \frac{1}{x}\,dx = dt \\ dx = x\cdot dt \end{array} \right\} = \int_2^{\infty} \frac{\cancel{x}\cdot dt}{\cancel{x}\cdot t} = \int_2^{\infty} \frac{dt}{t} = \ln|t| \Big|_2^{\infty} = \lim_{\beta\to\infty} \ln|t| \Big|_2^{\beta} = \infty$$

$$= \lim_{\beta\to\infty} (\ln(\ln|\beta|) - \ln(\ln|2|)) = \infty \Rightarrow \text{szereg jest rozbieżny}$$

29.12.2010r

$$\sum \frac{n+1}{n^2+3n+4} \leqslant \frac{n+n}{n^3} = \frac{2n}{n^3} = \sum 2\cdot\frac{1}{n^2} \rightarrow$$

majoranta: licznik ↑, mianownik ↓

minoranta: licznik ↓, mianownik ↑

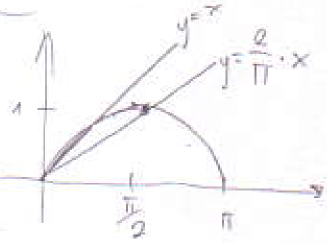
$$\frac{n}{n^2+3n^2+4n^2} \leqslant \frac{n+1}{n^2+3n+4}$$

$$\frac{n}{n^2(1+3+4)} = \frac{1}{8}\cdot\frac{1}{n} \rightarrow \text{szereg rozbieżny}$$

$$\sum \frac{n+1}{n^3+3n+4} \leqslant \frac{n+n}{n^3} \leqslant \frac{2n}{n^3} = 2\cdot\frac{1}{n^2} \quad \text{szereg geometryczny}$$

$$a_n = a_1\cdot\frac{1-q^n}{1-q} \qquad S_n = \frac{a_1}{1-q} \qquad q = \frac{a_{n+1}}{a_n}$$

$$\sum \sin\frac{1}{n} = \sum 2\sin\frac{1}{2n}\cdot\cos\frac{1}{2n}$$



02.01.2011r

$$\sum_{n=1}^{\infty} \frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{3^n\cdot n!} \qquad \lim_{n\to\infty}\frac{a_{n+1}}{a_n}$$

$$\lim_{n\to\infty} \frac{1\cdot3\cdot5\cdot(2n-1)\cdot(2(n+1)-1)}{3^{n+1}\cdot(n+1)!}\cdot\frac{3^n\cdot n!}{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)} =$$

$$= \lim_{n\to\infty}\frac{(2n+2-1)}{3^n\cdot3\cdot n!(n+1)}\cdot 3^n\cdot n! = \lim_{n\to\infty}\frac{2n+1}{3(n+1)} = \lim_{n\to\infty}\frac{2n+1}{3n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} =$$

$$= \lim_{n\to\infty}\frac{2+\frac{1}{n}}{3+\frac{3}{n}} = \lim_{n\to\infty}\frac{2+\frac{1}{n}}{3+\frac{1}{n}} = \frac{2}{3} < 1$$

Szereg $\sum_{n=1}^{\infty}\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{3^n\cdot n!}$ jest zbieżny o.k.

linia 1-przewodowa

$$L = \frac{\mu_0 l}{2\pi}\cdot\ln\frac{d}{r}\left[\frac{H}{m}\right]$$

$$C = \frac{2\pi}{\varepsilon\ln\frac{d}{r}}\left[\frac{F}{m}\right]$$

$$H = \frac{I}{2\pi r}$$

$$B = \mu\cdot H$$

$$\Phi = \int \vec{B}\cdot d\vec{S}$$

$$L = \frac{\Phi}{I}$$

03.01.2011r

$$\sum_{n=1}^{\infty}\frac{3^n}{(2n)!}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{3^{n+1}}{(2(n+1))!}\cdot\frac{(2n)!}{3^n} = \lim_{n\to\infty}\frac{3^n\cdot3}{2!\,n!(n+1)}\cdot\frac{2!\cdot n!}{3^n} = \lim_{n\to\infty}\frac{3}{n+1}\cdot\frac{\frac{1}{n}}{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{\frac{3}{n}}{1+\frac{1}{n}} = 0 \rightarrow \text{szereg jest zbieżny}$$

$$\frac{e}{2} > 1$$

$$\sum_{n=1}^{\infty}\frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n}$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n}} = \lim_{n\to\infty}\sqrt[n]{\frac{\left(1+\frac{1}{n}\right)^{n\cdot n}}{2^n}} = \lim_{n\to\infty}\frac{\sqrt[n]{\left(1+\frac{1}{n}\right)^{n\cdot n}}}{\sqrt[n]{2^n}} = \lim_{n\to\infty}\frac{e}{2} = \frac{2{,}718}{2}$$

szereg rozbieżny