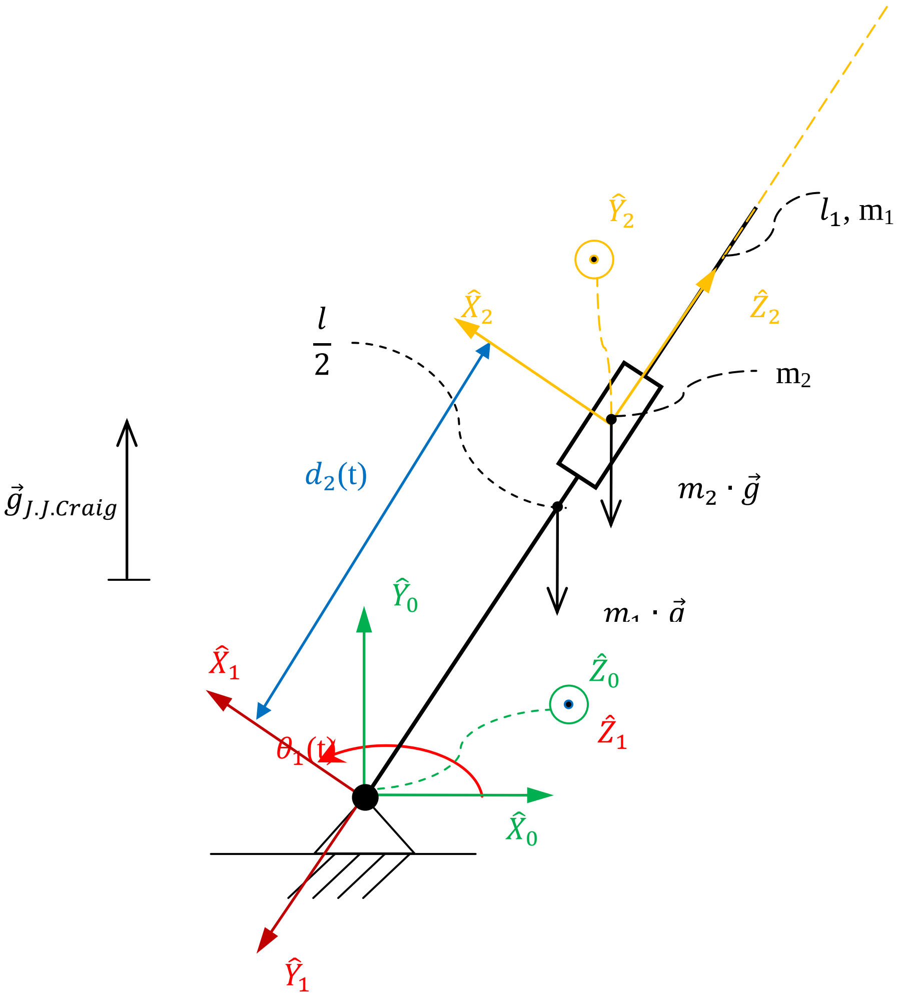
Dynamika robotów

http://www.mbmaster.pl



$$\left|\vec{g}_{J.J.Craig}\right| = |\vec{g}|$$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	$\theta_1(t)$
2	$\frac{\pi}{2}$	0	$d_2(t)$	0

$$ {}^{0}\underline{\omega}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

Dla i = 0

1)

$$ {}^{i+1}\underline{\omega}_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\underline{\omega}_i + \dot{\theta}_{i+1} \cdot {}^{i+1}\hat{z}_{i+1} $$

$$ {}^{1}\underline{\omega}_1 = {}^{1}_{0}R \cdot {}^{0}\underline{\omega}_0 + \dot{\theta}_1 \cdot {}^{1}\hat{z}_1 = \begin{bmatrix} c\theta_1 & s\theta_1 & 0 \\ -s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \dot{\theta}_1 \cdot {}^{1}\hat{z}_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} $$

2)

$$ {}^{i+1}\underline{\dot{\omega}}_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\underline{\dot{\omega}}_i + {}^{i+1}_{i}R \cdot {}^{i}\underline{\omega}_i \times \dot{\theta}_{i+1} \cdot {}^{i+1}\hat{z}_{i+1} + \ddot{\theta}_{i+1} \cdot {}^{i+1}\hat{z}_{i+1} $$

$$ {}^{1}\underline{\dot{\omega}}_1 = \underbrace{{}^{1}_{0}R \cdot {}^{0}\underline{\dot{\omega}}_0}_{0} + \underbrace{{}^{1}_{0}R \cdot {}^{0}\underline{\omega}_0}_{0} \times \dot{\theta}_1 \cdot {}^{1}\hat{z}_1 + \ddot{\theta}_1 \cdot {}^{1}\hat{z}_1 $$

$$ {}^{1}\underline{\dot{\omega}}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} $$

3)

$$ {}^{i+1}\underline{\dot{v}}_{i+1} = {}^{i+1}_{i}R \cdot \left[{}^{i}\underline{\dot{\omega}}_i \times {}^{i}\underline{P}_{i+1} + {}^{i}\underline{\omega}_i \times ({}^{i}\underline{\omega}_i \times {}^{i}\underline{P}_{i+1}) + {}^{i}\underline{\dot{v}}_i \right] $$

$$ {}^{1}\underline{\dot{v}}_1 = {}^{1}_{0}R \cdot \left[\underbrace{{}^{0}\underline{\dot{\omega}}_0 \times {}^{0}\underline{P}_1}_{0} + \underbrace{{}^{0}\underline{\omega}_0 \times ({}^{0}\underline{\omega}_0 \times {}^{0}\underline{P}_1)}_{0} + {}^{0}\underline{\dot{v}}_0 \right] $$

Założenie

$$^{1}\underline{\dot{v}}_{1} = \begin{bmatrix} g \cdot s\theta_1 \\ g \cdot c\theta_1 \\ 0 \end{bmatrix}$$

4)

$$^{i+1}\underline{\dot{v}}_{c_{i+1}} = {}^{i+1}\underline{\dot{\omega}}_{i+1} \times {}^{i+1}\underline{P}_{c_{i+1}} + {}^{i+1}\underline{\omega}_{i+1} \times ({}^{i+1}\underline{\omega}_{i+1} \times {}^{i+1}\underline{P}_{c_{i+1}}) + {}^{i+1}\underline{\dot{v}}_{i+1}$$

$$^{1}\underline{\dot{v}}_{c_1} = {}^{1}\underline{\dot{\omega}}_{1} \times {}^{1}\underline{P}_{c_1} + {}^{1}\underline{\omega}_{1} \times ({}^{1}\underline{\omega}_{1} \times {}^{1}\underline{P}_{c_1}) + {}^{1}\underline{\dot{v}}_{1}$$

$$^{1}\underline{\dot{v}}_{c_1} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -\frac{l_1}{2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \cdot \left(\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -\frac{l_1}{2} \\ 0 \end{bmatrix} \right) + \begin{bmatrix} g \cdot s\theta_1 \\ g \cdot c\theta_1 \\ 0 \end{bmatrix}$$

$$^{1}\underline{\dot{v}}_{c_1} = \begin{bmatrix} \frac{l_1}{2} \cdot \ddot{\theta}_1(t) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dot{\theta}_1^2(t) \cdot \frac{l_1}{2} \\ 0 \end{bmatrix} + \begin{bmatrix} g \cdot s\theta_1 \\ g \cdot c\theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} g \cdot s\theta_1 + \frac{l_1}{2} \cdot \ddot{\theta}_1(t) \\ g \cdot c\theta_1 - \frac{l_1}{2} \cdot \dot{\theta}_1^2(t) \\ 0 \end{bmatrix}$$

5)

$$^{i+1}\underline{F}_{i+1} = m_{i+1} \cdot {}^{i+1}\underline{\dot{v}}_{c_{i+1}}$$

$$^{1}\underline{F}_{1} = m_1 \cdot {}^{1}\underline{\dot{v}}_{c_1}$$

$$^{1}\underline{F}_{1} = \begin{bmatrix} m_1 \cdot \frac{l_1}{2} \cdot \ddot{\theta}_1(t) + m_1 \cdot g \cdot s\theta_1 \\ -m_1 \cdot \frac{l_1}{2} \cdot \dot{\theta}_1^2(t) + m_1 \cdot g \cdot c\theta_1 \\ 0 \end{bmatrix}$$

6)

$$ {}^{i+1}\underline{N}_{i+1} = {}^{i+1}\|_{c_{i+1}} \cdot {}^{i+1}\dot{\underline{\omega}}_{i+1} + {}^{i+1}\underline{\omega}_{i+1} \times {}^{i+1}\|_{c_{i+1}} \cdot {}^{i+1}\underline{\omega}_{i+1} $$

$$ {}^{1}\underline{N}_{1} = {}^{1}\|_{c_{1}} \cdot {}^{1}\dot{\underline{\omega}}_{1} + {}^{1}\underline{\omega}_{1} \times {}^{1}\|_{c_{1}} \cdot {}^{1}\underline{\omega}_{1} $$

$$ {}^{1}\|_{c_1} = \begin{bmatrix} I_{x_1} & -I_{x_1y_1} & -I_{x_1z_1} \\ -I_{x_1y_1} & I_{y_1} & -I_{y_1z_1} \\ -I_{x_1z_1} & -I_{y_1z_1} & I_{z_1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\cdot m_1 \cdot l_1^2 \end{bmatrix} $$

$$ {}^{1}\|_{c_1} \cdot {}^{1}\dot{\underline{\omega}}_{1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\cdot m_1 \cdot l_1^2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3}\cdot m_1 \cdot l_1^2 \cdot \ddot{\theta}_1(t) \end{bmatrix} $$

$$ {}^{1}\|_{c_1} \cdot {}^{1}\underline{\omega}_{1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3}\cdot m_1 \cdot l_1^2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3}\cdot m_1 \cdot l_1^2 \cdot \dot{\theta}_1(t) \end{bmatrix} $$

$$ {}^{1}\underline{\omega}_{1} \cdot {}^{1}\|_{c_1} \cdot {}^{1}\underline{\omega}_{1} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1(t) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3}\cdot m_1 \cdot l_1^2 \cdot \dot{\theta}_1(t) \end{bmatrix} = \begin{vmatrix} \hat{x}_1 & \hat{y}_1 & \hat{z}_1 \\ 0 & 0 & \dot{\theta}_1(t) \\ 0 & 0 & \frac{1}{3}\cdot m_1 \cdot l_1^2 \cdot \dot{\theta}_1(t) \end{vmatrix} = 0 $$

$$ {}^{1}\underline{N}_{1} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{3}\cdot l_1^2 \cdot \ddot{\theta}_1(t) \end{bmatrix} $$

Dla i = 1

$$ {}^1_2R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad {}^2_1R = {}^1_2R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} $$

1)

$$ {}^2\underline{\omega}_2 = {}^2_1R \cdot {}^1\underline{\omega}_1 + \dot{\theta}_2 \cdot {}^2\hat{z}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta}_1(t) \\ 0 \end{bmatrix} $$

2)

$$ {}^2\dot{\underline{\omega}}_2 = {}^2_1R \cdot {}^1\dot{\underline{\omega}}_1 \cdot \underbrace{{}^2_1R \cdot {}^1\dot{\underline{\omega}}_1 \times \dot{\theta}_2 \cdot {}^2\hat{z}_2}_{0} + \underbrace{\ddot{\theta}_2 \cdot {}^2\hat{z}_2}_{0} $$

$$ {}^2\dot{\underline{\omega}}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\ddot{\theta}_1(t) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \ddot{\theta}_1(t) \\ 0 \end{bmatrix} $$

3)

$$ {}^2\dot{\underline{v}}_2 = {}^2_1R \cdot \left[{}^1\dot{\underline{\omega}}_1 \times {}^1\underline{P}_2 + {}^1\underline{\omega}_1 \times ({}^1\underline{\omega}_1 \times {}^1\underline{P}_2) + {}^1\dot{\underline{v}}_1 \right] $$

$$ {}^2\dot{\underline{v}}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1(t) \end{bmatrix} \times \begin{bmatrix} 0 \\ -d_2(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1(t) \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1(t) \end{bmatrix} \times \begin{bmatrix} 0 \\ -d_2(t) \\ 0 \end{bmatrix} + \begin{bmatrix} g \cdot s\theta_1(t) \\ g \cdot c\theta_1(t) \\ 0 \end{bmatrix} \right) $$

$$^{2}\underline{\dot{v}}_{2}=\begin{bmatrix}\ddot{\theta}_1(t)\cdot d_2(t)\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\-\dot{\theta}_1^2(t)\cdot d_2(t)\end{bmatrix}+\begin{bmatrix}g\cdot s\theta_1\\0\\-g\cdot c\theta_1\end{bmatrix}$$

$$^{2}\underline{\dot{v}}_{2}=\begin{bmatrix}-\ddot{\theta}_1(t)\cdot d_2(t)+g\cdot s\theta_1\\0\\-\dot{\theta}_1^2(t)\cdot d_2(t)-g\cdot c\theta_1\end{bmatrix}$$

4)

$$^{2}\underline{\dot{v}}_{c_2}={}^{2}\underline{\dot{\omega}}_{2}\times{}^{2}\underline{P}_{c_2}+{}^{2}\underline{\omega}_{2}\times({}^{2}\underline{\omega}_{2}\times{}^{2}\underline{P}_{c_2})+{}^{2}\underline{\dot{v}}_{2}$$

$$^{2}\underline{\dot{v}}_{c_2}={}^{2}\underline{\dot{v}}_{2}$$

$$^{2}\underline{\dot{v}}_{c2}=\begin{bmatrix}-\ddot{\theta}_1(t)\cdot d_2(t)+g\cdot s\theta_1\\0\\-\dot{\theta}_1^2(t)\cdot d_2(t)-g\cdot c\theta_1\end{bmatrix}$$

5)

$$^{2}\underline{F}_{2}=m_2\cdot{}^{2}\underline{\dot{v}}_{c_2}$$

$$^{2}\underline{F}_{2}=\begin{bmatrix}m_2\cdot\ddot{\theta}_1(t)\cdot d_2(t)+m_2\cdot g\cdot s\theta_1\\0\\-m_2\cdot\dot{\theta}_1^2(t)\cdot d_2(t)-m_2\cdot g\cdot c\theta_1\end{bmatrix}$$

$$^{2}\underline{N}_{2}={}^{2}\|_{c_2}\cdot{}^{2}\underline{\dot{\omega}}_{2}+{}^{2}\underline{\omega}_{2}\times{}^{2}\|_{c_2}\cdot{}^{2}\underline{\omega}_{2}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Obliczenie siły napędowej dla układu „2"

$$\tau_2 = m_2 \cdot \ddot{d}_2(t) - m_2 \cdot \dot{\theta}_1^2(t) \cdot d_2(t) - m_2 \cdot g \cdot \cos\theta_1$$

Obliczenie momentu napędowego dla układu „1"

$$\tau_1 = \frac{1}{3} \cdot m_1 \cdot l_1^2 \cdot \ddot{\theta}_1(t) + \frac{1}{4} \cdot m_1 \cdot l_1^2 \cdot \ddot{\theta}_1(t) + m_1 \cdot g \cdot \sin\theta_1 \cdot \frac{l_1}{2} +$$
$$+ 2 \cdot m_2 \cdot d_2(t) \cdot \dot{\theta}_1(t) \cdot \dot{d}_2(t) + m_2 \cdot \ddot{\theta}_1(t) \cdot d_2^2(t) +$$
$$+ m_2 \cdot g \cdot \sin\theta_1(t) \cdot d_2(t) + m_2 \cdot \ddot{\theta}_1(t) \cdot d_2^2(t)$$